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Interface driven optimisation of springback in stretch bending of autobody panels

G.S. Hegde*, J. Shivaprasad, R. Sridhar
Department of Mechanical Engineering, R.V. College of Engineering,
Bangalore- 560059, India
* Corresponding author: E-mail address: gagehe@yahoo.co.in

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ABSTRACT

Purpose: Analytical determination of springback in the stretch bending of autobody double curved panels and optimization of parameters like yield strength and thickness of plate, have been the prime purposes of this paper.

Design/methodology/approach: The elasto plastic behaviour governed by Hooke's law and Holomann's law, has been solved using interface theory, the original contribution of author. In the process Hegde's interface modulus has been coined, which depends on both modulus of elasticity and the plastic coefficient.

Findings: The linearized functions of change of springback and stretch tension are solved for optimization, again using interface theory briefed in the Appendix.

Practical implications: The published models of springback optimization being complicated, a need has been felt to formulate computationally easy and confidently accurate model of springback determination. The theoretical deliberations are demonstrated on materials based on the data obtained from the published references.

Originality/value: The authors are confident of the originality and do claim authentically the novelty in the workout of the analysis. The interface theory applied at several stages in the analysis gives the optimal solutions to variables in the hyperspace defined by a linear system with redundancy in unknown variables.

Keywords: Springback; Optimization; Elsato-plastic relation; Interface theory

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ENGINEERING MATERIALS PROPERTIES

1. Introduction

In the tool design for forming operation of autobody panels made of sheet metal one of the decisive parameters to be considered is the spring back of the sheet. The shape of the formed metallic panel depends on the spring back, which is observed after removal of applied loads from the deformed sheet. The spring back is responsible for the deviation in the final deformed shape from the punch shape which has the desired shape of the product. The final unloaded configuration of the part includes internal deviations in the form of residual stresses and strains and external deviations in the form of localized thinning of the parts and errors in shape. Several researchers contributed towards the simple cases of spring back due to pure bending with and without stretching, on the theoretical and experimental basis. The analytical predictions had been reported in [1-3]. The problems in manufacturing because of springback in high strength steels, have been the recent topics of discussion in the works[4,5]. The spring back leads to curling problems, which has been dealt in [6]. The effect of restraining tension due to binder force on the control problems considering spring back had been discussed in [7]. The optimum binder force estimation to increase the drawability, to control the process parameters and to avoid the instability had been demonstrated through computer aided modeling approach in the literature [8]. The binder force control and shape aspects are treated separately [9] to evaluate the spring back to describe the shape perturbation in the paper [9]. Here emphasis is on the determination of the tool shape to yield correct part shape with the assumption of suitable binder condition. An in-process measurement is highlighted in [10]. In the literature [11], the estimation of spring back by in process measurement for single curved roll bending, had been demonstrated. In the works [12] the closed loop control philosophy was used to control the part shape by working out a transfer function relating the change in part shape to the change in the tool shape.

The estimation of the spring back of a double curved auto body panel based on the elasto-plastic deformation has been the prime objective of this paper. The complicated workout in [13] to evaluate the springback is simplified with the help of interface theory [14] which solves the elasto-plastic equation. The bidirectional stresses obtained are used to calculate the plastic stress and strain components. The plastic bending moment, hence obtained, aids in the calculation of the spring back and the tensions to be created by the binders are also obtained. Further the procedure of the parameter optimization is demonstrated which also utilizes the interface theory (explained briefly in appendix) to minimize the spring back under the constraint of tension force to be created by the binder. In simple terms, the interface theory gives the optimal values to the variables in a linear equation with redundancy in unknown variables. The material properties data of [13] have been used as reference in the numerical demonstrations of the theoretical deliberations of this effort.

The paper is organized in the following sections as, the background justification in introduction, formulation to give spring back calculation, numerical demonstration in implementation, comparison of results in discussion and finally the statement of the outcome in conclusion. The appendix briefly gives the interface theory for reference [14].

Interface theory being author's original contribution helps in solving the elasto-plastic stress strain relation appearing in the computational work. To the knowledge of authors the formulation methodology is new and different of its kind in the mechanics of sheet metal forming.

2. Formulation

The spring back causing geometrical surface defects in double curved autobody panels is initiated mainly because of the released plastic bending moments on unloading and non-uniform distribution of stress during forming. For understanding the later concept it is suggested to refer [15]. The former case is the focus of this study. For the elasto-plastic analysis the following assumptions are to be considered as, 1. There exists plane stress condition in the material so that $\sigma_3 = \sigma_z = 0$, and 2. The stress strain relation is characterized by the known conventional linearplot in the beginning and non-linear plot beyond yield. The material, within the yield region behaves linearly and obeys the Hooke's Law to be stated as

$$\sigma^e = E\varepsilon^e \tag{1}$$

where σ^e is the elastic stress, ε^e is the elastic strain and E is the modulus of elasticity. Beyond yield region, the material behaviour assumed is non- linear and this tends to obey the Holomann's law given by,

$$\sigma^p = K(\varepsilon^p)^n \tag{2}$$

where σ^p is the plastic stress, K is the work hardening modulus and *'n'* is the plastic index. ε^p is the plastic strain. It is true that when the processing behavior is not clear to be elastic or plastic it is wrong to assume the summation of Equations (1) and (2). But in the present case it is assumed that the forming by stretch bending has crossed the threshold of elasticity and processed in the complete plastic state. Hence the addition of Hooke's and Holomann's Equation is justified with the existing computational domain. The authors have checked this fact with reference to elasto-plastic analysis of pre-stressed holes. Hence from Equations (1) and (2) the total stress can be written as

$$\sigma = E\varepsilon^e + K(\varepsilon^p)^n \tag{3}$$

The Equation (3) being non linear, the solutions for ε^e and ε^p in terms of known σ, E, K and n, is not simple by computational methods available. But it is made easy in the application of interface theory (see Appendix) a specialized computational tool developed by author to handle linear system with more unknown than the number of equations at access. To obtain optimal values to ε^e and ε^p by interface theory it is imperative to arrange the terms in the ascending order of their co-efficient. Since $E > K$ for any material the Equation (3) is rearranged to

$$\sigma = K(\varepsilon^p)^n + E\varepsilon^e \tag{4}$$

The Equation (4) resembles with Equation (A1) of appendix with

$a_i = [K, E]$

$x_i = [(\varepsilon^p)^n, \varepsilon^e]$ and $b = \sigma$

Hence from interface theory

$$X_1 = (\varepsilon^p)^n = \frac{b}{a_2} = \frac{\sigma}{E} \text{ or } \varepsilon^p = \left(\frac{\sigma}{E}\right)^{1/n} \tag{5}$$

and

$$X_2 = \varepsilon^e = \left(1 - \frac{a_1}{a_2}\right)\frac{b}{a_2}$$

or, $$X_2 = \varepsilon^e = \left(1 - \frac{K}{E}\right)\frac{\sigma}{E} \tag{6}$$

From the generalized Hooke's law for a biaxial stress system the relations between stresses and strains in the x and y directions work out to be

$$\sigma_1^e = E\left(\varepsilon_1^e + \mu\,\varepsilon_2^e\right)/\left(1 - \mu^2\right) \tag{7}$$

where μ is the Poisson's ratio which is the ratio of ε_2 and ε_1, and

$$\sigma_2^e = E\left(\varepsilon_2^e + \mu\,\varepsilon_1^e\right)/\left(1 - \mu^2\right) \tag{8}$$

But from the Equation (6) it can be confirmed that

$$\varepsilon_1^e = \frac{\sigma_1}{E}\left(1 - \frac{K}{E}\right) \text{ or simply } (\lambda\,\sigma_1) \tag{9}$$

where $\lambda = \left(1 - \frac{K}{E}\right)\Big/E$ $1/\lambda$ = Hegde's interface modulus

and $\varepsilon_2^e = \lambda\,\sigma_2$

On substitution of (9) and (10) in expressions (7) and (8)

$$\sigma_1^e = \frac{E\lambda}{1 - \mu^2}\left(\sigma_1 + \mu\,\sigma_2\right) \tag{11}$$

$$\sigma_2^e = \frac{E\lambda}{1-\mu^2}(\sigma_2 + \mu\sigma_1) \tag{12}$$

As the elastic limit terminates at yield point, it can be reasonably assumed as $\sigma_1^e = \sigma_y$ = yield strength and hence

$$\sigma_y = A(\sigma_1 + \mu\sigma_2) \tag{13}$$

where $A = E\lambda/(1-\mu^2)$

By the application of interface theory to Equation (13) and since, $\mu A < A$

$$\sigma_1 = \frac{(1-\mu)\sigma_y}{A} \text{ and } \sigma_2 = \frac{\sigma_y}{A} \tag{14}$$

By substitution of solutions (14) in Equation (5) it is possible to arrive at

$$\varepsilon_1^p = \left(\frac{\sigma_1}{E}\right)^{\frac{1}{n}} = \left[\frac{(1-\mu)(1-\mu^2)\sigma_y}{E^2\lambda}\right]^{\frac{1}{n}} \tag{15}$$

$$\varepsilon_2^p = \left(\frac{\sigma_2}{E}\right)^{\frac{1}{n}} = \left[\frac{(1-\mu^2)\sigma_y}{E^2\lambda}\right]^{\frac{1}{n}} \tag{16}$$

Hence from the relations (15) and (16) the plastic stresses are

$$\sigma_1^p = K(\varepsilon_1^P)^n = \frac{K(1-\mu)(1-\mu^2)\sigma_y}{E^2\lambda} \tag{17}$$

$$\sigma_2^p = K(\varepsilon_2^P)^n = \frac{K(1-\mu^2)\sigma_y}{E^2\lambda} \tag{18}$$

2.1. Estimation of spring back

With high tensions in both directions, the neutral axis disappears from the cross section and both inner and outer surface of the panel deform plastically in tension which is depicted by the stress and strain distribution which is quite evident and well established fact. With this the forming of the panel exhibits an important phenomenon of tension controlled bending with purely plastic section of the panel.

On unloading of the punch, there result two changes- one is release of tension force leading to the sliding of sheet around the punch without any shape change and the other is release of plastic moment resulting in change of radius of curvature from punch radius R_p to sheet radius R_s.

It has been shown in reference [13] that

$$\Delta\left(\frac{1}{R}\right) = \left(\frac{1}{R_p} - \frac{1}{R_s}\right) = \frac{12M^p(1-\mu^2)}{Et^3} \tag{19}$$

However it is derived in paper [13] that the springback, Δh, with fairly good approximation

$$\Delta h = \frac{1}{2}\Delta\left(\frac{1}{R}\right)L^2 \tag{20}$$

where L is half the length of the panel in the intended direction as shown by Figure 1.

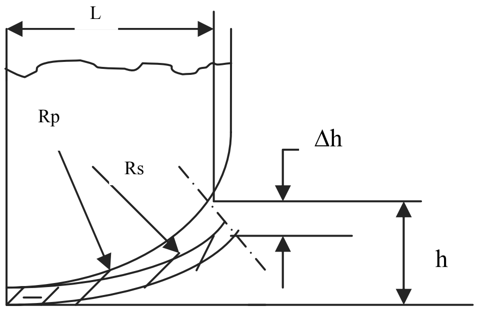

Fig. 1. Springback diagrams

From the mechanics of solids

$$M^p = \int_0^t \sigma^p Z dZ \tag{21}$$

$$= \frac{\sigma^p t^2}{2}$$

and $$T^p = \int_0^t \sigma^p dZ = (\sigma^p)t \tag{22}$$

The Equations (21) and (22) give the plastic moment and the tension in unit length of the panel. From relations (17), (19), (20) and (21) the spring back in direction '1' is

$$\Delta h_1 = \frac{1}{2}\left[\frac{12(1-\mu)(1-\mu^2)^2 k\sigma_y}{E^3 t\lambda}\right]L_1^2 \tag{23}$$

From relations (18), (19), (20) and (21) the spring back in the direction '2' is

$$\Delta h_2 = \left[\frac{6(1-\mu^2)^2 k\sigma_y}{E^3 t\lambda}\right]L_2^2 \tag{24}$$

The relations for tension to be applied in the two directions work out to be

$$T_1^p = \left[\frac{k(1-\mu)(1-\mu^2)\sigma_y}{E^2\lambda}\right]t \tag{25}$$

$$T_2^p = \left[\frac{k(1-\mu^2)\sigma_y}{E^2\lambda}\right]t \tag{26}$$

2.2. Springback minimization

For a panel of known and given length '$2L$' the spring back Δh is expressed by the function

$$\Delta h = f(\sigma_y, t) \tag{27}$$

The linearized expression for Equation (27) is

$$\Delta H = H_1\Delta\sigma_y + H_2\Delta t \tag{28}$$

where

$$H_1 = \frac{\partial H}{\partial \sigma_y} = \frac{6(1-\mu)(1-\mu^2)^2 k L_1^2}{E^3 t\lambda} \text{ for curvature 1}$$

or $= \frac{6(1-\mu^2)^2 k L_1^2}{E^3 t\lambda}$ for curvature 2

$$H_2 = \frac{\partial H}{\partial t} = -\frac{6(1-\mu)(1-\mu^2)^2 k \sigma_y L_2^2}{E^3 t^2 \lambda} \text{ for curvature 1}$$

or $= -\frac{6(1-\mu^2)^2 k \sigma_y L_2^2}{E^3 t^2 \lambda}$ for curvature 2

The tension in the panel is also expressed as a function of σ_y and t, as

$T = g(\sigma_y, t)$

The linearized Equation is

$$\Delta T = G_1 \Delta\sigma_y + G_2 \Delta t \tag{29}$$

where

$$G_1 = \frac{\partial T}{\partial \sigma_y} = \frac{k(1-\mu)(1-\mu^2)^2 t}{E^2 \lambda} \text{ for curvature 1}$$

or $= \frac{(1-\mu^2)^2 kt}{E^2 \lambda}$ for curvature 2

$$G_2 = \frac{\partial T}{\partial t} = \frac{k(1-\mu)(1-\mu^2)^2 \sigma_y}{E^2 \lambda} \text{ for curvature 1}$$

or $\frac{(1-\mu^2)^2 k\sigma_y}{E^2 \lambda}$ for curvature 2

The optimization problem is

Minimize $\Delta H = H_1 \Delta\sigma_y + H_2 \Delta t$ (30)

Subject to $G_1 \Delta\sigma_y + G_2 \Delta t = \Delta T$

where ΔT is the parameter, which is the constraint since $G_2 > G_1$

By the application of interface theory

$$\Delta\sigma_y = \frac{\Delta T}{G_2} \qquad \Delta t = \left(1 - \frac{G_1}{G_2}\right)\frac{\Delta T}{G_2}$$

Substituting this in (30)

$$\Delta H = \frac{H_1 \Delta T}{G_2} + H_2\left(1 - \frac{t}{\sigma_y}\right)\frac{\Delta T}{G_2} \tag{31}$$

$$\frac{H_1}{G_2} = \frac{6L^2}{Et^2} \text{ and } \frac{H_2}{G_2} = -\frac{6L^2}{Et^2}$$

Hence $-\Delta H = \frac{6L^2 \Delta T}{Et^2 \sigma_y}$ (32)

The Equation (32) gives the change in spring back for the change in tension force created by the binder force control. The change in spring back is the optimal one.

3. Implementation

With a purpose to demonstrate the theoretical findings from previous section, the panel with steel (FePO4) and 6016-t4-EDT, having the material data given in Table 1, have been used. The punch dimensions for the double curved auto panel are R_{p1} = 665 mm and R_{p2} = 2000 mm from reference [13] have been taken. The binder force is varied to obtain stretch tensions from 0, 100, 200, 300, 400 and 500 N. The spring back in both dimensions are computed as listed in Table 2.

Table 1.
Properties for material 1- Steel and material 2-6016-T4-EDT [13]

Sl.No	t(mm)	σ_y(MPa)	E(MPa)	K(MPa)	n	μ
1	0.8	172	205000	527	0.23	0.3
2	1.2	95	70000	344	0.24	0.33

Table 2.
Computed results

Parameters	Curvature 1		Curvature 2	
	Material 1	Material 2	Material 1	Material 2
ε^p	5.868E-15	1.343E-13	2.767E-14	7.12E-13
σ^p (MPa)	0.281	0.28	0.401	0.418
M^p(N-mm)	0.09	0.202	0.128	0.301
T^p(N)	0.2248	0.2424	0.1024	0.3612
Δh(mm)	0.062	0.118	0.2663	0.5322
L(mm)	115		200	
R_p(mm)	665		2000	

It is evident from Table 2 that the plastic strain increases with the length for a given material. There is appreciable change in the plastic stress with the curvature. For the same curvature the springback is dependent on the material elastic and plastic properties, never-the-less the geometric properties like thickness also play major role in the springback calculation. Brittle materials require higher tension and bending moment for stretch bending as they are more prone to springback effects. Larger the curvature length more is the springback observed. The tension force is a direct constraint in the reduction of springback. The materials with higher Young's modulus, higher yield strength and thickness produce less springback in the formed parts. Figure 2 shows the results of change in springback variations with respect to draw tension, for different materials with different curvature.

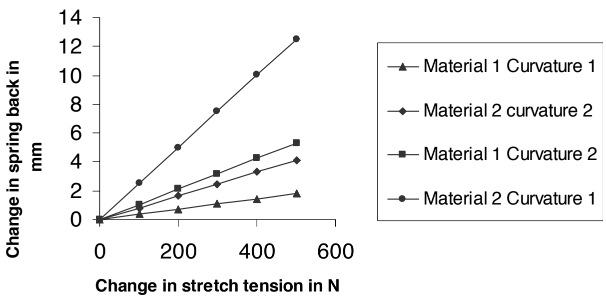

Fig. 2. Variation in spring back with stretch tension

3.1. Discussion

It is clearly evident that springback varies linearly with the yield strength. It projects that the materials of higher yield strength have higher spring back. Hence it can be concluded that the ductile materials have less spring back effect than the brittle materials. It can be observed that the spring back is inversely related to the thickness of the sheet used. By increasing the thickness it is possible to reduce the spring back. By stretching the sheet to higher tension using binder force control or a draw bead the bending renders to be purely plastic and the spring back is minimized. The spring back is also dependent on the length of the autobody panel, through direct relation to the square of the half length. It could not be neglected that the material properties like modulus of elasticity and the plastic modulus contribute well to the spring back effects. For a given material the stretch tension depends only on the thickness of the panel which is directly related. It may be noted that the spring back in the longitudinal direction is less than in transverse direction. The stretch tension in the longitudinal direction can be less compared to the transverse direction in the panel. The linearized model predicts the change in springback for a small change in thickness and the yield strength.

In the stretch bending it is assumed that the sheet slides over the tools as the punch is withdrawn. But with non-zero friction the sticking may occur at some point of forming process. Hence the assumption is invalid with the decreased value of binder force. Proper control over the binder force is required to overcome the sticking problem.

4. Conclusions

The purely plastic bending with stretching of auto body panel with double curvature has been modeled and the expression for spring back is presented. The elasto-plastic expression which is the combined result of Hooke's and Holomann's law, has been solved by interface theory, a new treatment tool that eases the process of solution. In the process the plastic stress, the plastic strain, the plastic moment and tension in the sheet are computed. Under high tension the sheet deforms by pure plastic bending. The spring back resulting on the release of plastic moment is estimated. The plane stress condition is assumed in the analysis. The optimization study to predict change in spring back under the constraint of change of tension has been presented. The interface theory is applied accompanied by the linearization. The theoretical deliberation is supported by the numerical implementation for the data accessed from the published literature. The methodology is entirely new and different from the earlier publications on the relevant topic. With the control over stretch tension the behaviour of spring back can be minimized. The spring back shows dependency over material strength and thickness for panel of given length. The biaxial stress strain, distribution in double curved auto body panel generalizes the formulation.

Appendix

Brief of interface theory [14]

The linear Equation with redundancy of unknown variables be,

$$\sum a_i X_i = b \tag{A1}$$

The simplest form for consideration of proof is,

$$a_1 X_1 + a_2 X_2 + a_3 X_3 = b \tag{A2}$$

The Equation (A2) can be written as $a_1 X_1 + V_1 = 0$ (A3)

where $V_1 = a_2 X_2 + a_3 X_3 - b$

The solutions to (A3) are $(X_1, V_1) = (h, -a_1 h)$

Hence, $a_2 X_2 + V_2 = -a_1 h$ (A4)

where $V_2 = a_3 X_3 - b$

The solutions to (A4) are, $(X_2, V_2) = \left[\left(1 - \frac{a_1}{a_2}\right)h, -a_2 h\right]$

Hence $a_3 X_3 + V_3 = -a_2 h$ (A5)

Solution to (A5) is, $(X_3, V_3) = \left[\left(1 - \frac{a_2}{a_3}\right)h, -a_3 h\right]$

But $V_3 = -b = -a_3 h, \quad or \quad h = \frac{b}{a_3}$

Hence $X_1 = \frac{b}{a_n}$

$$X_{1+1} = \left(1 - \frac{a_i}{a_{i+1}}\right) \times \left(\frac{b}{a_n}\right) \quad \text{for} \quad i = 1\ to\ (n-1)$$

From the solution to X_i it is evident that the maximization is effective when $a_i < a_{i+1}$ i.e, when the coefficients of the terms in (A1) are arranged in the ascending order of a_i. This concept is made use in elasto-plastic analysis in case of forming the sheet metals.

Nomenclature

a_i	Co-efficients in linear equation
b	Output specification constraint
E	Modulus of elasticity
G_1, G_2	Linearized co-efficients of tension
H_1, H_2	Linearized co-efficients of springback
K	Plastic Modulus
L	Half length of the sheet
M^p	Plastic Moment
n	Plastic index
R_p	Punch radius
R_s	Sheet radius
T	Tension force
t	Thickness of sheet
X_i	Interface variables
ΔH	Change in springback
Δh	Springback
σ^e	Elastic stress
σ^p	Plastic stress
ε^e	Elastic strain
ε^p	Plastic strain
λ	Hegde's interface modulus
μ	Poisson's ratio

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