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Hegde's instability mechanics for the prediction of forming limit in sheet metal forming

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ABSTRACT

Purpose: The purpose of the paper is to investigate the Hegde's instability mechanics for the prediction of forming limit in sheet metal forming.

Design/methodology/approach: Hegde's Instability Mechanics (HIM) paves way for explaining the effects of diffuse instability and localization due to necking in sheet metal forming. For different ranges of stress ratios, the ratio of strain differentials has been computed and hence the forming limits are predicted.

Findings: Basing the formulation of HIM on the von Misess yield criterion and applying the interface theory (briefed in appendix) the state-of- the-art purpose is deemed to be well served and demonstrated. Interface theory to explain in simple terms, gives the optimal decision variables in an 'n' dimensional hyper-space. The concept of HIM is demonstrated on isotropic and anisotropic materials. The anisotropic materials show better stability than isotropic materials in sheet metal forming. However the geometric instability is analyzed with the assumptions that the thickness stresses are negligible and biaxial state of stress persists in sheet metal forming.

Practical implications: The observations are based on the theoretical findings for which the experimental validation exists in the reviewed references.

Originality/value: To the sincere knowledge of authors, is both different and unique of its kind in sheet forming mechanics needing horizontal exploration by potential researchers.

Keywords: Sheet metal instability; Von-Misess yield criteria; Interface theory; Hegde's Instability Mechanics (HIM)

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MANUFACTURING AND PROCESSING OF ENGINEERING MATERIALS

1. Introduction

Relatively large imperfection is needed for the geometric instability in sheet metal forming. This fact has been dealt, by Azrin and Backofen [1]. The dynamic discontinuity with formation of a vertex [2] in the yield surface in the studies of instability had been proposed by Storen and Riee [2]. Three important findings [3] had been given in the researches done experimentally by the Anand et al. as 1. Shear band localization is preceded by plane–strain tension and diffuse necking, 2. For small positive rate of strain hardening, shear band localization can be expected, 3. Beyond the initiation point of shear band, considerable deformation can be sustained before fracture of materials. Shear instability in two modes, 1. Bulk-shear band, 2. Surface instability band have been investigated by Hutchinson and Tvergaard [4]. The forming limit diagrams of sheet metal and the influence of the plastic property of the material were of consideration in M-K theory [5]. Ghosh and Hecker [6] reported highest sensitivity of material imperfection in plane stretching than in out of plane stretching. Limit strain potential obtained by Chan *et al,* [7] had given an initial estimate by critical thickness strain criterion. Application of generalized criterion of Hill by Lian *et al.* [8] paved way for predicting sheet metal forming limit. Date and Padmanabhan [9] attempted to show that shear band to flow localization had been formed, irrespective of the presence or absence of imperfection.

The Swifts' diffuse instability and Hill's localized instability [10] are taken as the starting point in the contribution of this paper. In the absence of thickness stress von Misess yield criteria gives the effective stress, in the form of distortion energy equation of failure in terms of the two principal stresses. This has been solved using interface theory by Hegde *et al.* to optimize variables (differential change in principal stresses) to get the change in stain increments [11,12]. Further the load maximization condition in two principal directions gives the ratio of change in strain increments.

Levaillant and Chenot [13] predicated the defects in sheet metal forming being attributed to material process conditions. The domain knowledge base in the form of empirical rules for the industrial practice had been provided in by Graf and Hosford [14]. This gives substantial aid in design of metal forming tools and to plan the processes. The stable optimum solution to forming by finite element method (FEM) had been provided by Huh and Kim as the complication in formed shape needed analysis on computers [15].

The Swift's diffusing instability criteria states that instability is initiated when increment in the applied effective stress due to geometric softening exceeds the stress which can be produced by strain hardening. However it fails to explain the shift from diffuse instability to localization (necking) in the rate sensitive metals. The criterion due to Hill is applicable when one of the principal strains describing the deformation state is negative and other is positive. Both the diffuse and localization have the due explanation in effort of Hegde's Instability Mechanics (HIM).

HIM starting with distortion energy theory of failure, uses interface Theory (Briefed in appendix) as the mathematical treatment tool to optimize the ratio of change in strain increments for different ratios of principal stresses. By principle interface theory solves the redundant linear equation for the optimum values of the decision vector in the 'n' dimensional hyperspace defined by an indeterminate equation. The differentiation of the distortion energy failure equation results in one equation with two unknown variables with redundancy that creates suitable platform for application of interface theory, the original contribution in the computational mathematics as per the sincere claim of authors.

The paper is organized into introduction dealing with background survey; formulation giving the Hegde's Instability Mechanics (HIM); the implementation is presented in results and discussion; and the outcome of the paper in the form of conclusion. For brief reference of the interface theory the readers are provided with appendix. The authors are honestly confident of HIM results being original and different.

2. Formulation of HIM

In most of the sheet forming application, the thickness stress ($\sigma_3 = 0$) is quite small and can be neglected. The attention is focused to the plane stress case involving biaxial plain stress condition with principal stresses σ_1 and σ_2 being considered to obtain the increment in the effective stress (σ_e) using Von-Misess yield criteria. According to this distortions energy theory the instability is initiated when one or the other of the loads associated with the principal stresses passes through a maximum. The distortion energy theory being the most conservative one has the form.

$$\sigma_e = \left(\sigma_1^{\,2} - \sigma_1\sigma_2 + \sigma_2^{\,2}\right)^{\frac{1}{2}} \tag{1}$$

Differentiating the expression (1) leads to a form,

$$2\sigma_e = 2\sigma_1\frac{d\sigma_1}{d\sigma_e} - \sigma_1\frac{d\sigma_2}{d\sigma_e} - \sigma_2\frac{d\sigma_1}{d\sigma_e} + 2\sigma_2\frac{d\sigma_2}{d\sigma_e}$$

On manipulation, we arrive at,

$$2\sigma_e = \left(2\sigma_1 - \sigma_2\right)\frac{d\sigma_1}{d\sigma_e} + \left(2\sigma_2 - \sigma_1\right)\frac{d\sigma_2}{d\sigma_e} \tag{2}$$

The Equation (2) is an indeterminate Equation of the form of Equation (A1) of the appendix, with

$b = 2\sigma_e, a_i = \left[\left(2\sigma_1 - \sigma_2\right), \left(2\sigma_2 - \sigma_1\right)\right]$

And $X_i = \left[\dfrac{d\sigma_1}{d\sigma_e}, \dfrac{d\sigma_2}{d\sigma_e}\right]$

But application of optimal interface theory requires the condition

$a_i < a_{i+1}$ to be satisfied.

for $\sigma_1 > \sigma_2 \;\; a_1 > a_2 \;\; X_1 = \dfrac{d\sigma_2}{d\sigma_e} \;\; X_2 = \dfrac{d\sigma_1}{d\sigma_e}$

and for $\sigma_2 > \sigma_1 \;\; a_2 > a_1 \;\; X_2 = \dfrac{d\sigma_2}{d\sigma_e} \;\; X_1 = \dfrac{d\sigma_1}{d\sigma_e}$

Case 1:

By application of interface theory (refer appendix), for $\sigma_1 > \sigma_2$

$$\frac{d\sigma_2}{d\sigma_e} = \frac{2\sigma_e}{2\sigma_1 - \sigma_2} \tag{3}$$

And $$\frac{d\sigma_1}{d\sigma_e} = \left[1 - \left(\frac{2\sigma_2 - \sigma_1}{2\sigma_1 - \sigma_2}\right)\right]\frac{2\sigma_e}{\left(2\sigma_1 - \sigma_2\right)} \tag{4}$$

For load maximum along the '1' principal direction,

$$d\left(P_1\right) = d\left(\sigma_1 A_1\right) = \sigma_1 dA_1 + A_1 d\sigma_1 = 0 \tag{5}$$

This Equation (5) leads to

$$d\sigma_1 = \sigma_1 d\varepsilon_1 \tag{6}$$

Similarly, for load maximum along principal direction '2'

$$d\sigma_2 = \sigma_2 d\varepsilon_2 \tag{7}$$

From Equation (3) and (7)

$$d\sigma_2 = \sigma_2 d\varepsilon_2 = \frac{2\sigma_e d\sigma_e}{2\sigma_1 - \sigma_2} \tag{8}$$

From Equations (4) and (6)

$$d\sigma_1 = \left[1 - \frac{2\sigma_2 - \sigma_1}{2\sigma_1 - \sigma_2}\right]\frac{2\sigma_e d\sigma_e}{2\sigma_1 - \sigma_2} = \sigma_1 d\varepsilon_1 \tag{9}$$

By the manipulation of Equations (8) and (9), for, $\sigma_1 > \sigma_2$,the interface theory gives

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{\sigma_1}{\sigma_2} \Big/ \left[1 - \frac{(2\sigma_1 - \sigma_2)}{(2\sigma_2 - \sigma_1)}\right] \tag{10}$$

Case 2:

For $\sigma_2 > \sigma_1$ the interface theory gives

$$\frac{d\sigma_1}{d\sigma_e} = \frac{2\sigma_e}{(2\sigma_2 - \sigma_1)} \tag{11}$$

$$\frac{d\sigma_2}{d\sigma_e} = \left[1 - \frac{(2\sigma_1 - \sigma_2)}{(2\sigma_2 - \sigma_1)}\right]\frac{2\sigma_e}{(2\sigma_2 - \sigma_1)} \tag{12}$$

From Equations (6) and (11) and by manipulation

$$d\varepsilon_1 = \frac{1}{\sigma_1}\frac{(2\sigma_e d\sigma_e)}{(2\sigma_2 - \sigma_1)} \tag{13}$$

Using Equations (7) and (12) and after simple manipulation

$$d\varepsilon_2 = \frac{1}{\sigma_2}\left[1 - \frac{2\sigma_1 - \sigma_2}{2\sigma_2 - \sigma_1}\right]\frac{(2\sigma_e d\sigma_e)}{(2\sigma_2 - \sigma_1)} \tag{14}$$

By manipulating Equations (13) and (14),

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{\sigma_1}{\sigma_2}\left[1 - \frac{2\sigma_1 - \sigma_2}{2\sigma_2 - \sigma_1}\right] \tag{15}$$

By letting $\frac{\sigma_2}{\sigma_1} = X$ and from Equations (10) and (11) it is arrived at,

- For $\sigma_1 > \sigma_2$

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{1}{X} \Big/ \left[1 - \frac{(2X-1)}{(2-X)}\right] \tag{16}$$

- For $\sigma_2 > \sigma_1$

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{1}{X}\left[1 - \frac{2-X}{2X-1}\right] \tag{17}$$

By considering the Hill's anisotropic stress function as,

$$\sigma_e = \left(\sigma_1^2 - A\sigma_1\sigma_2 + \sigma_2^2\right)^{\frac{1}{2}} \tag{18}$$

Let $A = \frac{2R}{1+R}$

R is the plastic anisotropic parameter which is representative of mean of the property at different angles. Hence the ratio of strain increments is given by

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{1}{X} \Big/ \left[1 - \frac{2X - R}{2 - RX}\right] \qquad \text{For } \sigma_1 > \sigma_2 \tag{19}$$

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{1}{X}\left[1 - \frac{(2 - RX)}{(2X - R)}\right] \qquad \text{For } \sigma_2 > \sigma_1 \tag{20}$$

For isotropic material putting R = 1, the Equation (19) and (20) get reduced to Equations (16) and (17). According to Swift criterion [10]

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{R - 2X}{RX - 2} \tag{21}$$

3. Results and discussion

With the substitution of X values for isotropic materials (R = 1) in Equations (19) and (20), the values for the strain increment ratio is calculated as follows in Tables 1-6.

Table 1.
Strain increment ratio for $\sigma_1 > \sigma_2$, R = 1

X	0	0.2	0.5	0.6	0.8	0.9	1.0
$\frac{d\varepsilon_2}{d\varepsilon_1}$	$+\infty$	3.75	2.0	1.94	2.5	4.07	$+\infty$

Table 2.
Strain increments ratio for $\sigma_1 > \sigma_2$ and R= 0.89 (LPG steel)

X	0	0.2	0.5	0.6	0.8	0.9	1.0
$\frac{d\varepsilon_2}{d\varepsilon_1}$	$+\infty$	3.94	2.152	2.114	2.27	4.61	$+\infty$

Table 3.
Strain increment ratio for Swift criterion

X	0	0.2	0.5	0.6	0.8	0.9	1.0
$\frac{d\varepsilon_2}{d\varepsilon_1}$	-0.5	-0.269	0.0707	0.199	0.551	0.759	1.0

Table 4.
Strain increments when R = 1 and $\sigma_2 > \sigma_1$

X	1.1	1.2	1.5	1.6	1.8	1.9	2.0
$\frac{d\varepsilon_2}{d\varepsilon_1}$	0.23	0.325	0.5	0.511	0.512	0.507	0.5

Table 5.
Strain increments for $\sigma_2 > \sigma_1$, R = 0.89

X	1.1	1.2	1.5	1.6	1.8	1.9	2.0
$\frac{d\varepsilon_2}{d\varepsilon_1}$	0.2	0.32	0.41	0.47	0.473	0.47	0.464

Table 6.
Stability analysis using Equation (19); Strain increment ratio $\frac{d\varepsilon_2}{d\varepsilon_1}$ for $\sigma_1 > \sigma_2$

R \ σ_2/σ_1	0.6	0.7	0.8
0.5	2.833	3.14	4
0.6	2.63	2.89	3.65
0.7	2.438	2.66	3.33
0.8	2.26	2.45	3.035
0.9	2.09	2.25	2.76

In the uniaxial tension when $\sigma_2 = 0$ the stress ratio is zero and there is change in strain only in transverse direction of the sheet with no change in strain in longitudinal direction. This leads to infinite value to the ratio of differential strains. In the balanced biaxial tension, when $\sigma_2 = \sigma_1$ the change in strain also in the principal directions '1' and '2' are zero. These two explain the conditions of instability of diffusion and localization instability of the sheet metal forming. In the plane strain tension when $\sigma_2 = 0.6\sigma_1$ the ratio of strain increment is minimum as seen in Figures 1 and 2, that explains stability in forming process. The range of stability in anisotropic materials is better than that of isotropic materials as revealed by the comparison of plots in Figures 1 and 2. The mechanism of instability when $\sigma_1 > \sigma_2$ is numerically demonstrated in Table 1 for the isotropic materials and Table 2 for anisotropic material (LPG steel with R=0.89). The results are graphically depicted in Fig. 1 and Fig. 2 for isotropic and anisotropic materials respectively. When $\sigma_2 > \sigma_1$, the stability is evidenced between ratio of change strain increments (between 1.6 and 2 of change in stress increments) that are maximum for both isotropic and anisotropic materials. Beyond and within the stress change ratios 1.6 and 2.0 the instability observed prominent. The results for the second case is tabulated in Tables 4 and 5 and depicted in Fig. 4 and Fig. 5. The Table 3 and Fig. 3 projecting the results of Swift theory shows good agreement with localization, but a vague picture on diffuse instability. From the Equations derived in HIM it is not difficult to obtain the change in stress ratio for any change in strain increments ratio. Thus it can explain the Hill's localization condition also. This has the evidence in the extensions to Fig. 4 and Fig. 5. The results are from application useful, hypothetical and theoretical examples which confirm some of the experimental investigations in references [13-15].

Within the stress ratio 0.5 the load associated with σ_1 passes through maximum hence necking instability (localization) is anticipated. When stress ratio exceeds 0.6 and approaches 1.0 diffuse instability can be observed. This can be inferred from Fig. 1. For anisotropic materials the stability is preserved in the range of stress ratio 0.5-0.8 emphasizing that anisotropic materials exhibit better stability than isotropic materials, which is observed from Figures 1 and 2. From Fig. 3 it is clear that Swifts criteria fails to explain the localization (necking) instability but confirms the diffuse instability of HIM. Fig. 4 and Fig. 5 confirm both the instability criteria when $\sigma_2 > \sigma_1$ as observed oppositely in Fig. 1 and Fig. 2.

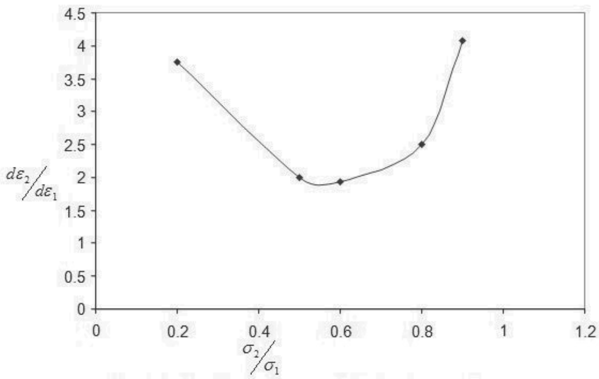

Fig. 1. Forming Limit Diagram for $\sigma_1 > \sigma_2$, R = 1

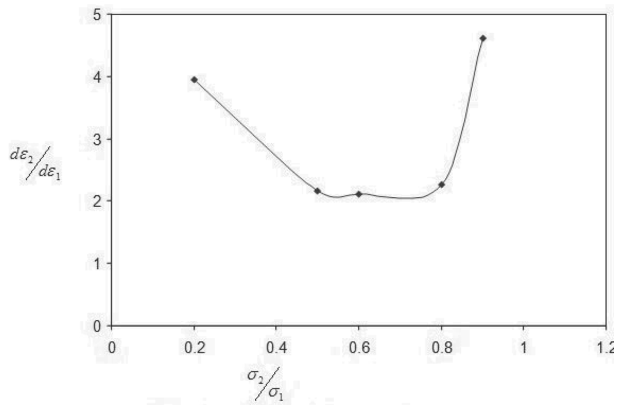

Fig. 2. Forming Limit Diagram for $\sigma_1 > \sigma_2$, R = 0.89

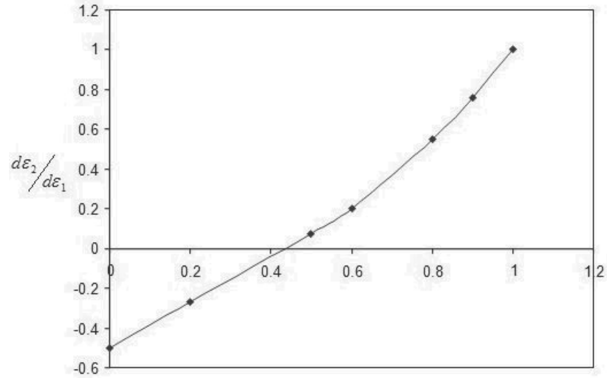

Fig. 3. Swift Criterion

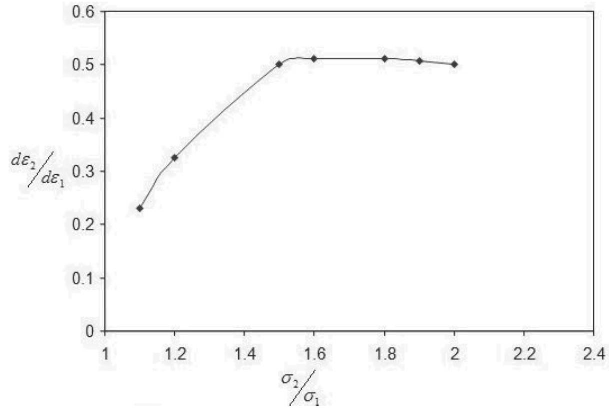

Fig. 4. Forming Limit Diagram for $\sigma_2 > \sigma_1$, R = 1

The experimental finding of Anand and Spitzig [3] is theoretically well justified from Fig. 1 and Fig. 2 as σ_1 increases from σ_2 onwards the stress ratio σ_2 / σ_1 goes on decreasing. Between the stress ratio 1 and 0.6 the shear band localization is observed before it passes through plane strain tension with $\sigma_2 = 0.6\sigma_1$. Further increase in σ_1 leading to stress ratio 0.6 to 0.2, the effect observed is diffuse necking which is evidenced by higher ratio of strain increment. In Fig. 1 the least ratio of strain increment is observed at stress ratio of 0.6.

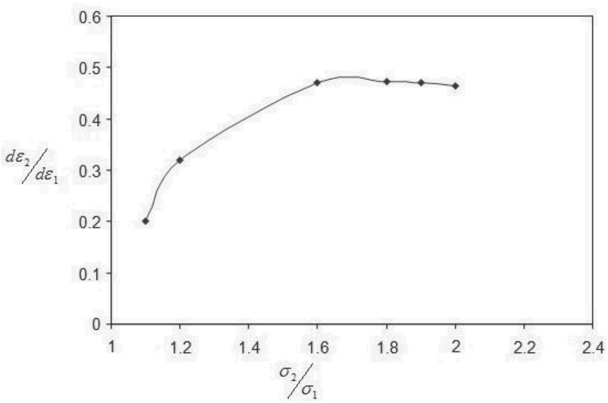

Fig. 5. Limit Diagram for $\sigma_2 > \sigma_1$, R = 0.89

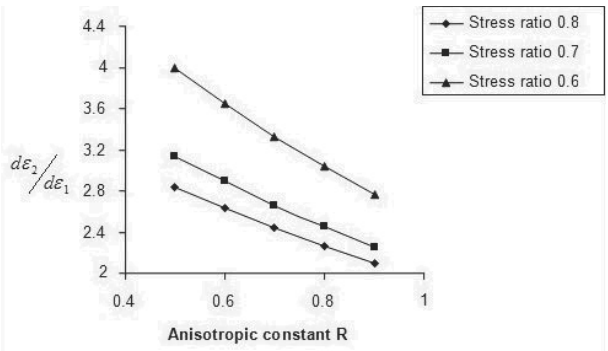

Fig. 6. Stability Analysis

Fig. 4 and Fig. 5 are forming limit diagrams obtained for $\sigma_2 > \sigma_1$. In the range of stress ratio 1.1 to 1.5 the ratio of strain increment keeps on increasing highlighting necking (localization) instability. The range of stress ratio (σ_2 / σ_1) 1.5 to 2 is the best for sheet metal forming from the point of view of stability beyond stress ratio 2 the diffuse instability is anticipated. Again it is a subtle point to note that anisotropic materials are more stable than isotropic materials in sheet metal forming. The stability of anisotropic materials with different is revealed in Table 6 and Figure 6.

4. Conclusions

Hegde's Instability Mechanics (HIM) is formulated to study the geometric instability in sheet metals made of isotropic and anisotropic materials. It highlights that the state of stress and anisotropy parameters are responsible for the geometric instability in sheet and Hill is accommodated to explain the mechanism by single theoretical concept HIM gives sufficient insight and explanations to experimental investigations reported by the earlier researches, Basing the start of analysis on the distortion energy theory and using interface theory as the treatment tool the optimal solution to the ratio of strain increments are achieved in HIM. Interface theory being authors' unique contribution works optimally to solve the redundant linear system to generate the unknown variables in the infinitely dimensioned hyper –space. The outcome of HIM shows a different turn to the angle at which the instability concept in sheet forming is looked at. Absolute perfection in the formulation of HIM is honestly claimed original to the knowledge of authors.

Appendix (brief of Interface theory)

The linear redundant Equation in general has the form,

$$\sum a_i X_i = b \tag{A1}$$

Here X $_i$ are the variables to be optimized based on the values of *b* and a_i. For the purpose of proof a specific form of (A1) with three terms is considered as

$$a_1 X_1 + a_2 X_2 + a_3 X_3 = b \tag{A2}$$

By the process of decoupling Equation (A2) can be rewritten as

$$a_1 X_1 + V_1 = 0 \tag{A3}$$

Where the decoupled segment, $V_1 = a_2 X_2 + a_3 X_3 - b$

The Equation (A3) is an indeterminate Equation for which the handle roots (initial roots) are

$(X_1, V_1) = (1, -a_1)$.

But Equation (A3) has infinite roots which are accomplished by proportionating handle roots by an interface adaptor h which is kept as common variable till the process of segmentation reaches the final step. In the final segment the h gets estimated to be the ratio of b and the coefficient of the last term in Equation (A1). The values of h are then substituted in the expressions for X$_{i.}$ Once the roots X$_i$ are obtained, the validity can be cross-checked by back substitution in Equation (A1).

The solutions to Equation (A3) are

$$(X_1, V_1) = (h, -a_1 h)$$

Now after decoupling $a_2 X_2 + V_2 = -a_1 h$ (A4)

The Equation (A4) has the solutions

$$(X_2, V_2) = \left[\left(1 - \frac{a_1}{a_2}\right)h, a_2 h\right]$$

Now $a_3 X_3 + V_3 = -a_2 h$ (A5)

The solutions to Equation (A5) are

$$(X_3, V_3) = \left[\left(1 - \frac{a_2}{a_3}\right)h, -a_3 h\right]$$

But $V_3 = -b = -a_3 h$, Hence $h = b/a_3$, The interface adaptor, *h* can take different values if the Equations (A3), (A4), and (A5) are different and not related. But they are the segments of Equation (A2). Hence interface adaptor has to be one and unique as can be verified by back-substitution of X_i in Equation (A2).

Hence to generalize the solutions to Equations (A1)

$$X_1 = h = \frac{b}{a_n}, X_{i+1} = \left(1 - \frac{a_i}{a_{i+1}}\right)\frac{b}{a_n} \text{ for i = 2 to } (n-1)$$

For X_i to be positive, *b*, a_n, and $\left(1 - \frac{a_i}{a_{i+1}}\right)$ are to be greater than zero.

It may be observed from the solutions to X_i that the positive optimal values are achieved when $a_i < a_{i+1,}$ (with b, and a_n are to be assumed positive) so that the co-efficient of the Equation (A1) are arranged in the ascending order of their magnitude. The same concept is made use in solving the Equation (2) of distortion energy theory of failure in this paper.

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