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System of second order robot arm problem by an efficient numerical integration algorithm

R. Ponalagusamy*, S. Senthilkumar
Department of Mathematics, National Institute of Technology,
Tiruchirappalli-620 015, Tamilnadu, India
* Corresponding author: E-mail address: rpalagu@nitt.edu

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ABSTRACT

Purpose: The aim of this article is focused on providing numerical solutions for system of second order robot arm problem using the Runge-Kutta Sixth order algorithm.

Design/methodology/approach: The parameters governing the arm model of a robot control problem have also been discussed through RK-sixth-order algorithm. The precised solution of the system of equations representing the arm model of a robot has been compared with the corresponding approximate solutions at different time intervals.

Findings: Results and comparison show the efficiency of the numerical integration algorithm based on the absolute error between the exact and approximate solutions. The stability polynomial for the test equation $\dot{\gamma}=\lambda\gamma$ (λ is a complex Number) using RK-butcher algorithm obtained by Murugesan et. al. [1] and Park et. al. [2,3] is not correct and the stability regions for RK-Butcher methods have been absurdly presented. They have made a blunder in determining the range for real parts of λh (h is a step size) involved in the test equation for RK-Butcher algorithms. Further, they have abruptly drawn the stability region for STWS method assuming that it is based on the Taylor's series technique.

Research limitations/implications: It is noticed that STWS algorithm is not based on the Taylor's series method and it is an A-stable method. In the present paper, a corrective measure has been taken to obtain the stability polynomial for the case of RK-Butcher algorithm, the ranges for the real part of λh and to present graphically the stability regions of the RK-Butcher methods.

Originality/value: Based on the numerical results and graphs, a thorough comparison is carried out between the numerical algorithms.

Keywords: RK-Sixth-Order algorithm; Ordinary differential equations; System of second order; Robot arm problem

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METHODS OF ANALYSIS AND MODELLING

1. Introduction

The dynamics of Robot arm problem was initially discussed by Taha [5]. Research in this area is still active and its applications are enormous. This is because of its nature of extending accuracy in the determination of approximate solutions and its flexibility. Many investigations [4-8] have analysed the various aspects of linear and non-linear systems.

Most of the Initial Value Problems (lVPs) are solved through Runge-Kutta (RK) techniques which in turn being applied to compute numerical solutions for variety of problems, which are modelled as and the differential equations are discussed by

Alexander and Coylc [11], Evans [12], Hung [13], Shampine and Watts [14,18]. Shampine and watts [12] have developed mathematical codes for the Runge-Kutta fourth order technique. Runge-Kutta formula of fifth order has been developed by Butcher [15-17]. The numerical solution of robot arm problem has been obtained [19]. The applications of Non-linear Differential–Algebraic Control Systems to Constrained Robot Systems have been discussed by Krishnan and Mcclamroch [21]. Also, Asymptotic observer design for Constrained Robot Systems have been analyzed by Huang and Tseng [9].

The rest of the article is organized as follows. Section 2 provides a notion about the basics of robot arm model problem with variable structure control and controller design. In section 3 the outline of RK-Sixth order technique is discussed with system of second order equations. Section 3 analyses in brief about the Numerical Algorithm for General1zed Linear State Space System. Finally discussion and conclusion is given in section 5.

2. Robot arm model and essential of variable structure

2.1. Robot arm model

It is well known that non-linearity and coupled characteristics are involved in designing a robot control system and its dynamic behavior. A set of coupled non-linear second order differential equations in the form of gravitational torques, Coriolis and Centrifugal forces represent the dynamics of the robot. The importance of the above three forces are dependent on the two physical parameters of the robot namely the load it carries and the speed at which the robot operates. The design of the control system becomes more complex when the end user needs more accuracy based on the variations of the parameters mentioned above. A detailed version of a robot's structure with proper explanation is given in [20]. Keeping the objective on solving the robot dynamic equations in real time computation in view, an efficient numerical technique is required. Taha [5] discussed about the dynamics of robot arm problem and it can be represented in the following form.

$$T = A(Q)\ddot{Q} + B(Q,\dot{Q}) + C(Q) \quad (1)$$

where $A(Q)$ is the coupled inertia matrix, $B(Q,\dot{Q})$ is the matrix of coriolis and centrifugal forces. $C(Q)$ is the gravity matrix, T is the input torques applied at various joints.

For a robot with two degrees of freedom, by considering lumped equivalent massless links, i.e. it means point load or in this case the mass is concentrated at the end of the links ,the dynamics are represented by

$$\begin{aligned} T_1 &= D_{11}\ddot{q}_1 + D_{12}\ddot{q}_{21} + D_{122}(\dot{q}_2)^2 + D_{112}(\dot{q}_1\dot{q}_2) + D_1 \\ T_2 &= D_{21}\ddot{q}_1 + D_{22}\ddot{q}_2 + D_{122}(\dot{q}_1)^2 + D_2 \end{aligned} \quad (2)$$

where

$$\begin{aligned}
D_{11} &= (M_1 + M_2)d_2^2 + 2M_2 d_1 d_2 \cos(q_2) \\
D_{12} &= M_2 d_2^2 + M_2 d_1 d_2 \cos(q_2) \\
D_{21} &= D_{12} \\
D_{22} &= M_2 d_2^2 \\
D_{112} &= -2M_2 d_1 d_2 \sin(q_2) \\
D_{122} &= -M_2 d_1 d_2 \sin(q_2) \\
D_{211} &= D_{122} \\
D_1 &= [(M_1 + M_2)d_1 \sin(q_1) + M_2 d_2 \sin(q_1 + q_2)]g \\
D_2 &= [M_2 d_2 \sin(q_1 + q_2)]g
\end{aligned}$$

The values of the robot parameters used are M1= 2Kg, M2 = 5Kg, d1 = d2 = 1m. In the case of problem of set point regulation the state vectors are represented as

$$X = (X_1, X_2, X_3, X_4)^T = (q_1 - q_{1d}, \dot{q}_1, q_2 - q_{2d}, \dot{q}_2)^T \quad (3)$$

where

q_1 *and* q_2 are the angles at joints 1 and 2 respectively, and q_1d *and* q_2d are constants. Hence, equation (2). may be written in state space representation as,

$$\begin{aligned}
\dot{e}_1 &= x_2 \\
\dot{x}_2 &= \frac{D_{22}}{d}(D_{122}X_2^2 + D_{112}X_2X_4 + D_1 + T_1) - \frac{D_{12}}{d}(D_{211}X_4^2 + D_2 + T_2) \\
\dot{e}_3 &= x_4 \\
\dot{x}_4 &= \frac{-D_{12}}{d}(D_{122}X_2^2 + D_{112}X_2X_4 + D_1 + T_1) - \frac{D_{12}}{d}(D_{211}X_4^2 + D_2 + T_2)
\end{aligned} \quad (4)$$

Here, the robot is simply a double inverted pendulum and the lagrangian approach is used to develop the equations.

It is observed that by selecting the suitable parameters, the non-linear equations (3) of the two-link robot-arm model may be reduced to the following system of linear equations [5] as

$$\begin{aligned}
\dot{e}_1 &= x_2 \\
\dot{x}_2 &= B_{10}T_1 - A_{11}x_2 - A_{10}e_1 \\
\dot{e}_3 &= x_4 \\
\dot{x}_4 &= B_{20}^2 T_2 - A_{21}^2 x_4 - A_{20}^2 e_3
\end{aligned} \quad (5)$$

From (5) one can obtain the system of second order linear equations

$$\ddot{x}_1 = -A_{11}\dot{x}_1 - A_{10}x_1 + B_{10}T_1$$

$$\ddot{x}_3 = -\dot{A}_{21}^2 x_3 - A_{20}^2 x_1 + B_{210}^2 T_2$$

where the values of the parameters concerning the joint-1 are given by,

$A_{10} = 0.1730$, $A_{11} = -0.2140$, $B_{10} = 0.00265$

and the values of parameters concerning the joint-2 are given by,

$A_{20} = 0.0438$, $A_{21} = 0.3610$, $B_{20} = 0.0967$

and by choosing $T_1 = 1$ and $T_2 = 1$ with initial conditions,

$[e_1(0) \quad x_2(0) \quad e_3(0) \quad x_4(0)]^T = [-1 \quad 0 \quad -1 \quad 0]^T$

and the corresponding exact solution is given by,

$$e_1(t)=e^{0.107t}[-1.15317919\cos(0.401934074t)+0.306991074\sin(0.401934074t)]+0.15317919$$

$$x_2(t)=e^{0.107t}[0.463502009\sin(0.401934074t)++0.123390173\cos(0.401934074t)]++e^{0.107t}[-1.15317919\cos(0.401934074t)++0.306991074\sin(0.401934074t)]$$

$$e_3(t)=1.029908976\quad e^{-0.113404416\ t}--6.904124484\quad e^{-0.016916839\ t}++4.874215508$$

$$x_4(t)=-0.116795962\,e^{-0.11340416t}++0.116795962\,e^{-0.016916389\,t} \tag{6}$$

3. Outline of RK-Sixth-order algorithm

System of second order linear differential equations originates in the form of mathematical formulation of problems in mechanics, electronic circuits, chemical process and electrical networks, etc. Hence, the concept of solving a second order equation using the RK-Sixth-order algorithm is extended to find the numerical solution of the system of second order equations as given below. It is of importance to mention that one has to determine the upper limit of the step-size (h) in order to have a stable numerical solution of the given ordinary differential equation with IVP. Keeping this in view,

Consider the system of second order Initial Value Problems,

$$\ddot{y}_j=f_j(x,y_j,\dot{y}_j),\ j=1,2,.....m \tag{7}$$

with $y_j(x_0)=y_{j0}$

$\dot{y}_j(x_0)=\dot{y}_{j0}$ for all j = 1,2,…..m

Then the RK-Sixth-Order algorithm to determine y_j and $\dot{y}_j, j=1,2,3,....m$ are given by,

$$y_{jn+1}=y_{jn}+h\,[\frac{13}{200}k_{1j}+\frac{11}{40}k_{3j}+\frac{11}{40}k_{4j}+\frac{4}{25}k_{5j}+\frac{4}{25}k_{6j}+\frac{13}{200}k_{7j}] \tag{8}$$

and

$$\dot{y}_{jn+1}=\dot{y}_{jn}+h[\frac{13}{200}u_{1j}+\frac{11}{40}u_{3j}+\frac{11}{40}u_{4j}+\frac{4}{25}u_{51j}+\frac{4}{25}u_{6j}+\frac{13}{200}u_{7j}]$$

$$k_{1j}=\dot{y}_{jn},$$

$$k_{2j}=\dot{y}_{jn}+\frac{hu_{1j}}{2},$$

$$k_{3j}=\dot{y}_{jn}+\frac{2hu_{1j}}{9}+\frac{4hu_{2j}}{9},$$

$$k_{4j}=\dot{y}_{jn}+\frac{7hu_{1j}}{36}+\frac{2hu_{2j}}{9}-\frac{hu_{3j}}{12}$$

$$k_{5j}=\dot{y}_{jn}-\frac{35hu_{1j}}{144}-\frac{55hu_{2j}}{36}+\frac{35hu_{3j}}{48}+\frac{15hu_{4j}}{8},$$

$$k_{6j}=\dot{y}_{jn}-\frac{hu_{1j}}{360}-\frac{11hu_{2j}}{36}-\frac{hu_{3j}}{8}+\frac{hu_{4j}}{2}+\frac{hu_{5j}}{10},$$

$$k_{7j}=\dot{y}_n-\frac{41hu_{1j}}{260}+\frac{22hu_{2j}}{13}+\frac{43hu_{3j}}{156}--\frac{118hu_{4j}}{39}+\frac{32hu_{5j}}{195}+\frac{80hu_{6j}}{39} \tag{9}$$

$$u_{1j}=f(x_n,y_{jn},\dot{y}_{jn}),$$

$$u_{2j}=f(x_n+\frac{h}{2},y_{1n}+\frac{hk_{11}}{2},y_{2n}+\frac{hk_{12}}{2},...,y_{mn}++\frac{hk_{1m}}{2},\dot{y}_{1n}+\frac{hu_{11}}{2},\dot{y}_{2n}+\frac{hu_{12}}{2},...,\dot{y}_{mn}+\frac{hu_{1m}}{2}),$$

$$u_{3j}=f(x_n+\frac{2h}{3},y_{1n}+\frac{2hk_{11}}{9}+\frac{4hk_{21}}{9},y_{2n}+\frac{2hk_{12}}{9}+\frac{4hk_{22}}{9},...,y_{mn}+\frac{2hk_{1m}}{9},+\frac{4hk_{2m}}{9},\dot{y}_{1n}+\frac{2hu_{11}}{9}+\frac{4hu_{21}}{9},\dot{y}_{2n}+\frac{2hu_{12}}{9}+\frac{4hu_{22}}{9},...,\dot{y}_{mn}+\frac{2hu_{1m}}{9}+\frac{4hu_{2m}}{9}),$$

$$u_{4j}=f(x_n+\frac{h}{3},y_{1n}+\frac{7hk_{11}}{36}+\frac{2hk_{21}}{9}-\frac{hk_{31}}{12},y_{2n}+\frac{7hk_{12}}{36}+\frac{2hk_{22}}{9}-\frac{2hk_{32}}{9},...,y_{mn}+\frac{7hk_{1m}}{36},+\frac{2hk_{2m}}{9}-\frac{hk_{3m}}{12},,\dot{y}_{1n}+\frac{7hu_{11}}{36}+\frac{2hu_{21}}{9}-\frac{hk_{31}}{12},]\dot{y}_{2n}+\frac{7hu_{12}}{36}+\frac{2hu_{22}}{9}-\frac{2hk_{32}}{9},...,\dot{y}_{mn}+\frac{7hu_{1m}}{36}+\frac{2hu_{2m}}{9}-\frac{hk_{3m}}{12}),$$

$$u_{5j}=f(x_n+\frac{5h}{6},y_{1n}-\frac{35hk_{11}}{144}-\frac{55hk_{21}}{36}+\frac{35hk_{31}}{48}+\frac{15hk_{41}}{8},$$
$$y_{2n}-\frac{35hk_{12}}{144}-\frac{55hk_{22}}{36}+\frac{35hk_{32}}{48}+\frac{15hk_{42}}{8},...,y_{mn}$$
$$-\frac{35hk_{1m}}{144}-\frac{55hk_{2m}}{36}+\frac{35hk_{3m}}{48}+\frac{15hk_{4m}}{8},$$
$$\dot{y}_{1n}-\frac{35hu_{11}}{144}-\frac{55hu_{21}}{36}+\frac{35hu_{31}}{48}+\frac{15hu_{41}}{8},$$
$$\dot{y}_{2n}-\frac{35hu_{12}}{144}-\frac{55hu_{22}}{36}+\frac{35hu_{32}}{48}+\frac{15hu_{42}}{8},....,$$
$$\dot{y}_{mn}-\frac{35hu_{1m}}{144}-\frac{55hu_{2m}}{36}+\frac{35hu_{3m}}{48}+\frac{15hu_{4m}}{8})$$

$$u_{6j}=f(x_n+\frac{h}{6},y_{1n}-\frac{hk_{11}}{360}-\frac{11hk_{21}}{36}-\frac{hk_{31}}{8}+\frac{hk_{41}}{2}+\frac{hk_{51}}{10},$$
$$y_{2n}-\frac{hk_{12}}{360}-\frac{11hk_{22}}{36}-\frac{hk_{32}}{8}+\frac{hk_{42}}{2}+\frac{hk_{52}}{10},...,y_{mn}$$
$$-\frac{hk_{13}}{360}-\frac{11hk_{23}}{36}-\frac{hk_{33}}{8}+\frac{hk_{43}}{2}+\frac{hk_{53}}{10},$$
$$\dot{y}_{1n}-\frac{hu_{11}}{360}-\frac{11hu_{21}}{36}-\frac{hu_{31}}{8}+\frac{hu_{41}}{2}+\frac{hu_{51}}{10},$$
$$\dot{y}_{2n}-\frac{hu_{12}}{360}-\frac{11hu_{22}}{36}-\frac{hu_{32}}{8}+\frac{hu_{42}}{2}+\frac{hu_{52}}{10},....,$$
$$\dot{y}_{mn}-\frac{hu_{13}}{360}-\frac{11hu_{23}}{36}-\frac{hu_{33}}{8}+\frac{hu_{43}}{2}+\frac{hu_{53}}{10})$$

$\forall\ j=1,2,3....,m$

$$u_{7j}=f(x_n+h,y_{1n}-\frac{41hk_{11}}{260}+\frac{41hk_{21}}{13}+\frac{43hk_{31}}{156}-\frac{118hk_{41}}{39}+$$
$$+\frac{32hk_{51}}{195}+\frac{80hk_{61}}{39},y_{2n}-\frac{41hk_{12}}{260}+\frac{41hk_{22}}{13}+\frac{43hk_{32}}{156}-$$
$$-\frac{118hk_{42}}{39}+\frac{32hk_{52}}{195}+\frac{80hk_{62}}{39},...,y_{mn}-\frac{41hk_{1m}}{260}+\frac{41hk_{2m}}{13}+$$
$$+\frac{43hk_{3m}}{156}-\frac{118hk_{4m}}{39}+\frac{32hk_{5m}}{195}+\frac{80hk_{6m}}{39},$$
$$\dot{y}_{1n}-\frac{41hu_{11}}{260}+\frac{41hu_{21}}{13}+\frac{43hu_{31}}{156}-\frac{118hu_{41}}{39}+\frac{32hu_{51}}{195}+$$
$$+\frac{80hu_{61}}{39},\dot{y}_{2n}-\frac{41hu_{12}}{260}+\frac{41hu_{22}}{13}+\frac{43hu_{32}}{156}-\frac{118hu_{42}}{39}+$$
$$+\frac{32hu_{52}}{195}+\frac{80hu_{62}}{39},....,\dot{y}_{mn}-\frac{41hu_{1m}}{260}+\frac{41hu_{2m}}{13}+\frac{43hu_{3m}}{156}-$$
$$-\frac{118hu_{4m}}{39}+\frac{32hu_{5m}}{195}+\frac{80hu_{6m}}{39}) \quad (10)$$

The corresponding RK-Sixth order array to represent Equation (9) takes form as follows:

0							
$\frac{1}{2}$	$\frac{1}{2}$						
$\frac{2}{3}$	$\frac{2}{9}$	$\frac{4}{9}$					
$\frac{1}{3}$	$\frac{7}{36}$	$\frac{2}{9}$	$\frac{-1}{12}$				
$\frac{5}{6}$	$\frac{-35}{144}$	$\frac{-55}{36}$	$\frac{35}{48}$	$\frac{15}{8}$			
$\frac{1}{6}$	$\frac{-1}{360}$	$\frac{-11}{36}$	$\frac{-1}{8}$	$\frac{1}{2}$	$\frac{1}{10}$		
1	$\frac{-41}{260}$	$\frac{22}{13}$	$\frac{43}{156}$	$\frac{-118}{39}$	$\frac{32}{195}$	$\frac{80}{39}$	
	$\frac{13}{200}$	0	$\frac{11}{40}$	$\frac{11}{40}$	$\frac{4}{25}$	$\frac{4}{25}$	$\frac{13}{200}$

Therefore, the final integration is a weighted sum of the six calculated derivatives and the RK-sixth-order predictor formula is given by,

$$y_{n+1}=y_n+[\frac{13}{200}k_1+\frac{11}{40}k_3+\frac{11}{40}k_4+\frac{4}{25}k_5+\frac{4}{25}k_6+\frac{13}{200}k_7] \quad (11)$$

Substituting the expressions of k_1, k_2, k_3, k_4, k_5, k_6 and k_7 into equation (11) we get,

$$y_{n+1}=y_n+\frac{h\lambda y_n}{2160}[2160+2160z+1080z^2+360z^3+ \quad (12)$$
$$+90z^4+18z^5+3z^6-z^7]$$

From equation (12), the stability of the polynomial Q(z) = y_{n+1}/y_n becomes,

$$Q(z)=1+z+\frac{z^2}{2}+\frac{z^3}{6}+\frac{z^4}{24}+\frac{z^5}{120}+\frac{z^6}{720}-\frac{z^7}{2160} \quad (13)$$

In a similar manner, the stability polynomial for the test equation $\dot{y}=\lambda y$ (λ is a complex constant) using the RK-Butcher technique has been obtained as

$$Q_1(z)=1+z+\frac{z^2}{2}+\frac{z^3}{6}+\frac{z^4}{24}+\frac{z^5}{120}+\frac{z^6}{640} \quad (14)$$

At this juncture, it is pertinent to point out that Murugesan et. al.[1] and Park et. al.[2,3] have obtained incorrect stability polynomial for the same test equation by adapting the RK-Butcher technique and these are respectively given by

$$Q_2(z)=1+z+\frac{z^2}{2}+\frac{z^3}{6}+\frac{z^4}{24}+\frac{z^5}{120}+\frac{z^6}{720} \quad (15)$$

$$Q_3(z)=1+z+\frac{z^2}{2}+\frac{z^3}{6}+\frac{z^4}{360}-\frac{853z^5}{19008} \quad (16)$$

It is of importance to mention that Murugesan et. al. [1] and Park et.al.[2,3] have not presented the mathematical derivation of obtaining the stability polynomial for single term walsh series (STWS) technique. The stability region for STWS technique shown in papers published by them is not similar to the one for RK-Butcher technique which is based on the Taylor series method. A careful study shows that STWS technique is not based on the Taylor series technique but it is an A-stable technique.

Keeping this in view, the form of stability region for STWS technique is ridicules and incorrect. Moreover, Murugesan et. al. [1] and Park et.al.[2,3] have made a wrong comparative analysis of the stability region of the RK-Butcher technique using an incorrect version of the stability polynomials (Equations (9)(15) and (10)(16)) obtained by them. Further, the stability region of the STWS technique is abruptly assumed by them.

Also, they have made a critical mistakes to be determined i.e., the range for the real part of λh in the cases of RKAM and the RK-Butcher techniques. The wrong range for the real part of λh is $-2.780 < Re(z) < 0.0$ in the RK-Butcher technique. Similar types of severe mistakes have been detected in the paper authored by the same group (Sekar et. al. [4]). In view of this, we have presented the corrected version of the stability region of the RK-Butcher technique which are shown in Figure 2.

In this stability regions, the range for the real part of λh is $-3.463 < Re(z) < 0.0$ in the RK-Butcher algorithm.

4. Results and conclusion

The discrete and exact solutions of the robot arm model problem have been computed for different time intervals using the equations (5) and (9) which are depicted in Tables 1-4. The values of $e_1(t)$, $x_2(t)$,$e_3(t)$ and $x_4(t)$ are calculated for time t arranging from 0.25 to 1. The absolute error between the exact and discrete solutions for the RK techniques based on RK-Fifth-order and RK-Sixth-order are calculated. For time t = 0.0, 0.25, 0.05, 0.75 and 1.0 the values are tabulated in Tables 1-4 respectively.

It is significant to stress that the obtained discrete solutions for the Robot Arm model problem using the RK-Sixth-order algorithm guarantees more accurate values in comparison with RK-Fifth-order technique. It is of interest to mention that the stability region for STWS technique drawn by Murugesan et. al [1] and Park et. al. [2,3] is not correct. At this juncture, the present authors have made an observation that the stability region for STWS technique presented by them is not correct owing to the reason that STWS technique is not based on the Taylor series technique and it is the type of A-stable technique. The numerical solutions calculated using RK-Sixth-order algorithm are in very close to the exact solutions of the robot arm model problem while the RK-Fifth Order technique gives rise to a considerable error. Therefore, RK-Sixth-order algorithm is more suitable for studying the system of second order robot arm model problem and this algorithm can be implemented for any length of independent variable on a digital computer.

Table 1.
Solutions of Equations (3) and (15) for $x_1(t)$

Sol. No.	Time	Exact Solution	RKAM Solution	RKAM Error	RK-Butcher Solution	RK-Butcher Error	RK-Sixth Order Solution	RK-Sixth Order Error
1	0.00	-1.0000000000	-1.0000000000	0.0000000000	-1.0000000000	0.0000000000	-1.0000000000	0.0000000000
2	0.25	-0.9936586212	-0.9953327587	-0.0016741375	-0.9953327583	-0.0016741371	-0.9953327581	-0.0016741370
3	0.50	-0.9742424794	-0.9786473848	-0.0044049054	-0.9786473825	-0.0044049031	-0.9743245899	-0.0000821105
4	0.75	-0.9412482065	-0.9494398199	-0.0081916134	-0.9494352670	-0.0081870605	-0.9451413296	-0.0038931231
5	1.00	-0.8942959125	-0.9073316683	-0.0130357558	-0.9073036550	-0.0130077425	-0.9030901483	-0.0087942358

Table 2.
Solutions of Equations (3) and (15) for $x_2(t)$

Sol.	Time	Exact Solution	RKAM Solution	RKAM Error	RK-Butcher Solution	RK-Butcher Error	RK-Sixth Order Solution	RK-Sixth Order Error
1	0.00	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
2	0.25	0.0511410611	0.0459864489	0.0051546122	0.0459864413	0.0051546198	0.0459863413	0.0051546098
3	0.50	0.1045249896	0.0941265859	0.0104450	0.0941265909	0.0104450	0.0940922750	0.0051546101
4	0.75	0.1596829669	0.1438971697	0.0157857972	0.1439868497	0.0156961172	0.1439950296	0.0156879373
5	1.00	0.2161001218	0.1949942351	0.0211058867	0.1950883237	0.0210117981	0.1966502520	0.0194498698

Table 3.
Solutions of Equations (3) and (15) for $x_3(t)$

Sol. No.	Time	Exact Solution	RKAM Solution	RKAM Error	RK-Butcher Solution	RK-Butcher Error	RK-Sixth Order Solution	RK-Sixth Order Error
1	0.00	-1.00000 00000	-1.00000 00000	0.00000 00000	-1.00000 00000	0.00000 00000	-1.00000 00000	0.00000 00000
2	0.25	-0.99965 16946	-0.99973 51600	-0.00008 34654	-0.99970 57337	0.00005 40391	-0.99970 56599	-0.00005 39653
3	0.50	-0.99862 16177	-0.99871 98532	0.00009 82355	-0.99869 04291	0.00006 88114	-0.99869 04092	-0.00006 87915
4	0.75	-0.99693 17452	-0.99700 73822	0.00007 56370	-0.99697 79638	0.00004 62186	-0.99696 69638	-0.00003 52186
5	1.00	-0.99460 34264	-0.99462 09249	0.00001 74985	-0.99461 95155	0.00001 60891	-0.99461 60056	0.00001 25792

Table 4.
Solutions of Equations (3) and (15) for $x_4(t)$

Sol. No.	Time	Exact Solution	RKAM Solution	RKAM Error	RK-Butcher Solution	RK-Butcher Error	RK-Sixth Order Solution	RK-Sixth Order Error
1	0.00	0.00000 00000	0.00000 00000	0.00000 00000	0.00000 00000	0.00000 00000	0.00000 00000	0.00000 00000
2	0.25	0.00277 18839	0.00285 05791	-0.00007 86952	0.00285 05764	-0.00007 86925	0.00284 99524	-0.00007 80685
3	0.50	0.00545 45872	0.00560 69156	-0.00015 23284	0.00560 68988	-0.00015 23116	0.00560 62955	-0.00015 17083
4	0.75	0.00805 06523	0.00879 39398	-0.00074 32875	0.00827 17292	-0.00022 10769	0.00827 11480	-0.00022 04957
5	1.00	0.01056 25499	0.01084 77411	-0.00028 51912	0.01084 75497	-0.00028 49998	0.01084 71859	-0.00028 4636

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