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Model for a piezoelectric strip of crack arrest subjected to Mode-I loadings

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ABSTRACT

Purpose: In the present paper a crack arrest model is proposed for an infinitely long narrow poled piezoelectric strip embedded with a centrally situated finite hairline straight crack.

Design/methodology/approach: The ceramic of the strip is assumed to be mechanically brittle and electrically ductile. Combined mechanical and electrical loads applied at the edge of the strip open the rims of the crack in mode-I deformations. Consequently a yield zone protrudes ahead of each tip of the crack. Under small scale yielding the yield zone are assumed to lie on the line segment along the axis of the crack. To arrest the crack from further opening the rims of the yield zones are subjected to normal, cohesive quadratically varying yield point stress. Two cases are considered: Case-I the edges of the strip are subjected to in-plane normal and in-plane electrical displacement and in Case-II the in-plane stresses and in-plane electrical field are applied on the edges of the strip. In each case problem is solved using Fourier transform method which finally reduces to the solution of integral equation.

Findings: Analytic expressions are derived for stress intensity factor, yield zone, crack opening displacement, crack growth rate, variation of these quantities with respect to affecting parameters viz. width of the strip, yield zone length, crack length, material constants for different ceramics have been plotted.

Research limitations/implications: The material of the strip is assumed mechanically brittle and electrically ductile consequently mechanically singularity is encountered first. The investigations in this paper are carried at this level. Also the crack yielding under the loads is considered small scale hence the yield zone is assumed to be lying on the line segment ahead of the crack.

Practical implications: Piezoelectric ceramics are widely used as sensors and actuators, this necessity prompts the fracture study on such ceramics under different loading conditions.

Originality/value: The paper gives an assessment of the quadratically varying load required to be prescribed on yield zones so as to arrest the opening of the crack. The investigations are useful to smart material design technology where sensors and actuators are manufactured.

Keywords: Smart materials; Stress intensity factor; Yield zone; Crack opening displacement; Crack growth rate

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ENGINEERING MATERIALS

1. Introduction

The work on piezoelectric strip weakened by a crack was started by Ozawa, Nowacki and Shindo by calculating the singular stresses and electric fields of a cracked piezoelectric strip [1]. The work was further [2] extended to dynamic analysis of a cracked piezoelectric material. A computer simulation of a high velocity impact experiment with aluminum projectile and ceramic cylinder rod has been carried in [3]. Fourier transform method together with linear theory of piezoelectricity is utilized [4] for calculating the singular stresses and electric field in an orthotropic piezoelectric ceramic strip containing a Giffith crack under longitudinal shear. Their study also investigates [5] the electro-elastic intensification near anti-plane shear crack in orthotropic piezoelectric ceramic strip. Crack tip field of an infinite piezoelectric strip under anti-plane impact is dealt in [6]. A shear zone model with parallel boundaries is used to evaluate the dynamic cutting forces in orthogonal cutting [7]. A fourth power stress intensity factor crack growth equation for an orthotropic piezoelectric ceramic strip is developed [8]. Under Mode-III loading for a straight crack symmetrically situated and oriented in a direction parallel to edges of strip, the dynamic electro-mechanical response of a piezoelectric strip with a constant crack vertical to the boundary is investigated [9] based on the superposition and integral transform technique. The linear piezoelectricity theory is applied to investigate the dynamic response of a centrally situated crack perpendicular to the edges of the piezoelectric strip subjected to anti-plane mechanical and electrical impacts [10]. A crack growth rate equation is found [11] for a finite crack in a narrow transversely isotropic piezoelectric ceramic body under tensile loading based on yield strip method , the solution is found using integral transform method. Fracture behavior of cracked poled piezoelectric material strip under combined mechanical and electrical loads is investigated [12] when the crack is vertical to the top and bottom edges of the strip. The saturation strip model for piezoelectric crack is re-examined [13] in a permeable environment to analyze fracture toughness of a piezoelectric ceramic for a permeable crack. In this paper [14] the dynamic anti-plane problem for a functionally graded piezoelectric strip containing an impermeable /permeable central crack vertical to the boundary is investigated using integral transforms and dislocation density function to reduce the problem to Cauchy singular integral equations. Generalized Dugdale model solution is presented in [15] for a piezoelectric plate weakened by two straight cracks. Fracture resistance behavior of Alumina-Zirconia composites is studied in [16]. A plane strain problem for an interface crack with an artificial contact zone near its tips, along the fixed edge of a piezoelectric semi-infinite strip under concentrated electro-mechanical loading is examined [17] by using Fourier transforms, boundary integral relations are derived. The electro-elastic behavior of a Griffith crack in a functionally graded piezoelectric strip is investigated [18] assuming that the stiffness, piezoelectric constant and dielectric permittivity of the functionally graded piezoelectric strip vary continuously as an exponential function and that the strip is under out-of-plane mechanical loading and in-plane electrical loading. A crack arrest model is proposed [19] for a poled piezoelectric plate weakened by a straight crack. The crack opens due to the tension at infinity consequently the plastic zones are developed which are closed by with variable yield point stresses. The mixed mode crack problem for a functionally graded piezoelectric strip is considered [20] assuming that the electro elastic properties of the strip vary continuously along the thickness of the strip and that the strip is under in-plane electric loading [21].

2. Mathematical formulation

Plane strains problem in xoz plane is defined as:

$$u_x = u(x,z)\ ,u_z = w(x,z)\ \text{and}\ u_y = 0, \tag{1}$$

$$E_x = E_x(x,z) = -\phi_{,x},\ E_z = E_z(x,z) = -\phi_{,z}\ , \quad E_y = 0;\ E_i = -\phi_{,i} \tag{2}$$

where u_i and E_i $(i = x, y, z)$ are the displacement and electrical field components respectively. Electric potential is denoted by ϕ and a comma denotes the partial derivative with respect to argument following it.

Governing equations for transversely isotropic piezoelectric ceramics may be written as:

$$\begin{pmatrix} c_{11}\frac{\partial^2}{\partial x^2}+c_{44}\frac{\partial^2}{\partial z^2} & (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z} & (e_{31}+e_{15})\frac{\partial^2}{\partial x\partial z} \\ (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z} & c_{44}\frac{\partial^2}{\partial x^2}+c_{33}\frac{\partial^2}{\partial z^2} & e_{15}\frac{\partial^2}{\partial x^2}+e_{33}\frac{\partial^2}{\partial z^2} \\ (e_{31}+e_{15})\frac{\partial^2}{\partial x\partial z} & e_{15}\frac{\partial^2}{\partial x^2}+e_{33}\frac{\partial^2}{\partial z^2} & -\varepsilon_{11}\frac{\partial^2}{\partial x^2}-\varepsilon_{33}\frac{\partial^2}{\partial z^2} \end{pmatrix} \begin{pmatrix} u_x \\ u_z \\ \phi \end{pmatrix} = 0 \tag{3}$$

where c_{11}, c_{13}, c_{33} and c_{44} are elastic constants; e_{31}, e_{33} and e_{15} are electric constants ; ε_{11} and ε_{33} stand for dielectric constants.

Solution of equation (3) using Fourier transforms may be written as:

$$u(x,z) = -\frac{2}{\pi}\sum_{j=1}^{3} a_j \int_0^\infty [A_j(\alpha)\sinh(\gamma_j\alpha z) + B_j(\alpha)\cosh(\gamma_j\alpha z)]\sin(\alpha x)\,d\alpha, \tag{4}$$

$$w(x,z) = \frac{2}{\pi}\sum_{j=1}^{3}\frac{1}{\gamma_j}\int_0^\infty [A_j(\alpha)\cosh(\gamma_j\alpha z) + B_j(\alpha)\sinh(\gamma_j\alpha z)]\cos(\alpha x)\,d\alpha + a_h z. \tag{5}$$

$$\phi(x,z) = -\frac{2}{\pi}\sum_{j=1}^{3}\frac{b_j}{\gamma_j}\int_0^\infty [A_j(\alpha)\cosh(\gamma_j\alpha z) + B_j(\alpha)\sinh(\gamma_j\alpha z)]\cos(\alpha x)\,d\alpha - b_h z, \tag{6}$$

where $A_j(\alpha)$ and $B_j(\alpha)$ are arbitrary functions to be determined ; a_h, b_h are constants to be determined from edge conditions on the strip and γ_j^2 $(j = 1,2,3)$ are the roots of characteristic equation.

$$A\gamma^6 - B\gamma^4 + C\gamma^2 - D = 0, \tag{7}$$

where:

$$A = -c_{44}(c_{33}\,\varepsilon_{33} + e_{33}{}^2),$$

$$\begin{aligned} B = &-2c_{44}\,e_{15}e_{33} - c_{11}\,e_{33}{}^2 - c_{33}(c_{44}\,\varepsilon_{11} \\ &+ c_{11}\,\varepsilon_{33}) + \varepsilon_{33}(c_{13}+c_{14})^2 + 2e_{33}(c_{13} \\ &+ c_{14})(e_{31}+e_{15}) - c_{44}{}^2\,\varepsilon_{33} - c_{33}(e_{31}+e_{15})^2, \end{aligned}$$

$$\begin{aligned} C = &-2c_{11}e_{15}e_{33} - c_{44}e_{15}{}^2 - c_{11}(c_{33}\varepsilon_{11} + c_{44}\varepsilon_{33}) \\ &+ \varepsilon_{11}(c_{13}+c_{44})^2 - 2e_{15}(c_{13}+c_{44}) \\ &\times (e_{31}+e_{15}) - c_{44}{}^2\,\varepsilon_{11} - c_{44}(e_{31}+e_{15})^2, \end{aligned} \tag{8}$$

$$D = -c_{11}(c_{44}\,\varepsilon_{11} + e_{15}{}^2),$$

The constant a_j and b_j are given by:

$$\begin{aligned} a_j = &\{(e_{31}+e_{15})(c_{33}\,\gamma_j{}^2 - c_{44}) - (c_{13}+c_{44}) \\ &\times (e_{33}\gamma_j{}^2 - e_{15})\}/\{(c_{44}\gamma_j{}^2 - c_{11}) \\ &\times (e_{33}\,\gamma_j{}^2 - e_{15}) + (c_{13}+c_{44})(e_{31}+e_{15})\gamma_j{}^2\}, \end{aligned} \tag{9}$$

$$b_j = \frac{(c_{44}\,\gamma_j{}^2 - c_{11})a_j + (c_{13}+c_{44})}{e_{31}+e_{15}}. \tag{10}$$

As is well-known the governing equation (3) in vacuum reduces to Laplace equation:

$$\nabla^2\phi = 0. \tag{11}$$

where ∇^2 is the two-dimensional Laplacian operator.

Taking Fourier transform of equation (11) the electric potential in vacuum $\phi^V(x,z)$ is then, written as:

$$\phi^V(x,z) = \frac{2}{\pi}\int_0^\infty A_4(\alpha)\sinh(\alpha z)\cos(\alpha x)\,d\alpha\ ,$$

for $0 \le x < c$, (12)

where $A_4(\alpha)$ is the unknown to be determined. The constitutive equation for electric displacement and electric field may be written as:

$$D_x = \varepsilon_0 E_x = -\varepsilon_0\phi_{,x}\,, \qquad D_z = \varepsilon_0 E_z = -\varepsilon_0\phi_{,z}\,, \tag{13}$$

while for transversely isotropic piezoelectric material these may be written as:

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{zz}\\ \sigma_{xz}\\ D_x\\ D_z\end{pmatrix} = \begin{pmatrix} c_{11} & c_{13} & 0 & 0 & -e_{31}\\ c_{13} & c_{33} & 0 & 0 & -e_{33}\\ 0 & 0 & c_{44} & -e_{15} & 0\\ 0 & 0 & e_{15} & \varepsilon_{11} & 0\\ e_{31} & e_{33} & 0 & 0 & \varepsilon_{33}\end{pmatrix}\begin{pmatrix}u_{,x}\\ w_{,z}\\ w_{,x}+u_{,z}\\ E_x\\ E_z\end{pmatrix}, \tag{14}$$

Opening mode stress intensity factor $K_I(a)$ at the tip x=a is defined as:

$$K_I(a) = \lim_{x\to a^+}\left\{[2\pi(x-a)]^{1/2}\ \sigma_{zz}(x,0)\right\}. \tag{15}$$

Crack opening displacement $\delta(x)$ is defined by:

$$\delta(x) = 2w(x) = \frac{4}{F_1}\int_x^a m(x,\alpha)K_I(\alpha)\,d\alpha, \tag{16}$$

where $m(x,\alpha)$ is given by:

$$m(x,\alpha) = \sqrt{\frac{\alpha}{\pi}}\ \frac{1}{\sqrt{\alpha^2 - x^2}}. \tag{17}$$

3. The problem

An infinitely long, narrow width (2h) transversely isotropic piezoelectric strip with hexagonal symmetry occupies oxyz region. The strip is poled along oz-direction and is embedded with a finite hairline straight crack, L, lying in the interval $|x| < c$ on ox-axis. The edges of the strip are subjected to uniform normal constant stress field $\sigma_{zz} = \sigma_h$ and Case-I: uniform normal electric displacement $D_z = D_0$ or Case-II: uniform constant normal electrical field $E_z = E_0$ consequently the rims of the crack open in mode-I type deformations forming a yield zone ahead of each tip of crack. Under small scale yielding the yield zones are assumed to lie along the line segment ahead of the crack length and occupy the region $c \le |x| \le a$. To arrest the crack from further opening the rims of the developed yield zones are subjected quadratically varying yield point stress. A schematic presentation of the configuration is given below in Figure 1.

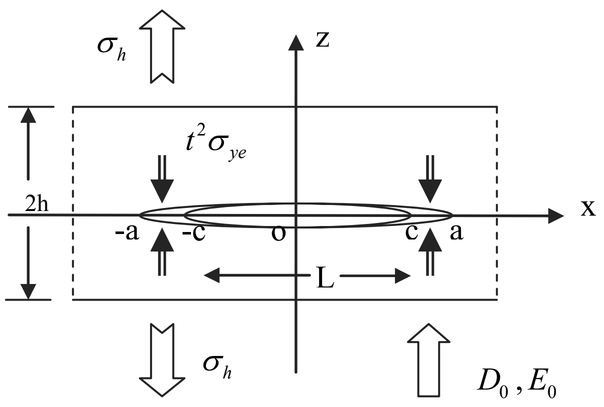

Fig. 1. Schematic presentation of the problem

4. Mathematical model

The strip under consideration is weakened by a straight crack of length 2a {= [-a,-c]∪ [-c,c] ∪ [c,a]} lying on ox-axis. The configuration is subjected to following conditions:

(a) $\sigma_{zx}(x,0)=0\,,\sigma_{zx}(x,h)=0,$ for $0\le x<\infty$,

(b) $\sigma_{zz}(x,0)=\dfrac{x^2}{c^2}\sigma_{ye}\,H(x-c),$ for $0\le x\le a$,

(c) $u_z(x,0)=0,$ for $a\le x<\infty$,

(d) $\phi(x,0)=0,$ for $c\le x<\infty$,

(e) $D_z(x,0)=D^V{}_z(x,0)\,,E_x(x,0)=E^V{}_x(x,0),$ for $0\le x<c$

where $H(x-c)$ is the Heaviside-step function and superscript 'V' denotes the quantities refer to vacuum inside the crack.

5. Solution of the problem

5.1. Case-I

The desired quantities are written from equations (4, 5, 6) in which unknown A_j and B_j are determined using boundary condition (ei).

$A_1(\alpha)=\gamma_1(b_2 f_3-b_3 f_2)C(\alpha)/d\,,\quad A_2(\alpha)=\gamma_2(b_3 f_1-b_1 f_3)C(\alpha)/d$

$$A_3(\alpha)=-\frac{\gamma_3}{f_3}\left(\frac{f_1}{\gamma_1}A_1(\alpha)+\frac{f_2}{\gamma_2}A_2(\alpha)\right), \tag{18}$$

$$B_1(\alpha)=\{-k_1(\alpha)A_1(\alpha)+l_1(\alpha)A_2(\alpha)\}/F(\alpha), \tag{19}$$

$$B_2(\alpha)=(k_2(\alpha)A_1(\alpha)-l_2(\alpha)A_2(\alpha))\,/\,F(\alpha), \tag{20}$$

$$B_3(\alpha)=(k_3(\alpha)A_1(\alpha)+l_3(\alpha)A_2(\alpha))\,/\,F(\alpha), \tag{21}$$

where:

$$f_j=c_{44}(a_j\gamma^2{}_j+1)-b_j e_{15},\; g_j=c_{13}a_j-c_{33}+e_{33}b_j,$$
$$h_j=e_{31}a_j-e_{33}-\varepsilon_{33}b_j\,, \tag{22}$$

and k_j and $l_j\;(j=1,\,2,\,3)$ denote:

$$k_1(\alpha)=\frac{f_1(g_2h_3-g_3h_2)}{\gamma_1}\cosh(\gamma_2\alpha h)\times[\cosh(\gamma_1\alpha h)\cosh(\gamma_3\alpha h)-1]+\frac{f_2(g_3h_1-g_1h_3)}{\gamma_2}\sinh(\gamma_1\alpha h)\times\sinh(\gamma_2\alpha h)\cosh(\gamma_3\alpha h)+\frac{f_3(g_1h_2-g_2h_1)}{\gamma_3}\sinh(\gamma_1\alpha h)\times\cosh(\gamma_2\alpha h)\sinh(\gamma_3\alpha h),$$

$$k_2(\alpha)=\frac{f_1(g_3h_1-g_1h_3)}{\gamma_1}[\cosh(\gamma_1\alpha h)-\cosh(\gamma_3\alpha h)],$$

$$k_3(\alpha)=\frac{\gamma_3 f_1}{\gamma_1 f_3}\left\{\frac{f_1(g_2h_3-g_3h_2)}{\gamma_1}\sinh(\gamma_1\alpha h)\times\cosh(\gamma_2\alpha h)\sinh(\gamma_3\alpha h)+\frac{f_2(g_3h_1-g_1h_3)}{\gamma_2}\cosh(\gamma_1\alpha h)\times\sinh(\gamma_2\alpha h)\sinh(\gamma_3\alpha h)+\frac{f_3(g_1h_2-g_2h_1)}{\gamma_3}\cosh(\gamma_2\alpha h)\times[\cosh(\gamma_1\alpha h)\cosh(\gamma_3\alpha h)-1]\right\}. \tag{23}$$

$$l_1(\alpha)=\frac{f_2(g_2h_3-g_3h_2)}{\gamma_2}[\cosh(\gamma_2\alpha h)-\cosh(\gamma_3\alpha h)],$$

$$l_2(\alpha)=\frac{f_1(g_2h_3-g_3h_2)}{\gamma_1}\sinh(\gamma_1\alpha h)\times\sinh(\gamma_2\alpha h)\cosh(\gamma_3\alpha h)+\frac{f_2(g_3h_1-g_1h_3)}{\gamma_2}\cosh(\gamma_1\alpha h)\times[\cosh(\gamma_2\alpha h)\cosh(\gamma_3\alpha h)-1]+\frac{f_3(g_1h_2-g_2h_1)}{\gamma_3}\cosh(\gamma_1\alpha h)\times\sinh(\gamma_2\alpha h)\sinh(\gamma_3\alpha h),$$

$$l_3(\alpha)=\frac{\gamma_3 f_2}{\gamma_2 f_3}\left\{\frac{f_1(g_2h_3-g_3h_2)}{\gamma_1}\sinh(\gamma_1\alpha h)\times\cosh(\gamma_2\alpha h)\sinh(\gamma_3\alpha h)+\frac{f_2(g_3h_1-g_1h_3)}{\gamma_2}\cosh(\gamma_1\alpha h)\times\sinh(\gamma_2\alpha h)\sinh(\gamma_3\alpha h)+\frac{f_3(g_1h_2-g_2h_1)}{\gamma_3}\cosh(\gamma_1\alpha h)\times[\cosh(\gamma_2\alpha h)\cosh(\gamma_3\alpha h)-1].\right. \tag{24}$$

and:

$$F(\alpha)=\frac{f_1(g_2h_3-g_3h_2)}{\gamma_1}\sinh(\gamma_1\alpha h)\times\cosh(\gamma_2\alpha h)\cosh(\gamma_3\alpha h)+\frac{f_2(g_3h_1-g_1h_3)}{\gamma_2}\cosh(\gamma_1\alpha h)\times\sinh(\gamma_2\alpha h)\cosh(\gamma_3\alpha h)+\frac{f_3(g_1h_2-g_2h_1)}{\gamma_3}\cosh(\gamma_1\alpha h)\times\cosh(\gamma_2\alpha h)\sinh(\gamma_3\alpha h)\,. \tag{25}$$

The new unknown $C(\alpha)$ introduced is determined, using boundary conditions (b and c), from the pair of integral equations

$$\frac{2}{\pi}\int_0^\infty \alpha\, G_1(\alpha) C(\alpha)\cos\alpha x\, d\alpha = -\sigma_h + \frac{x^2}{c^2}\sigma_{ye} H(x-c);\ 0 \le x < a \qquad (26)$$

$$\int_0^\infty C(\alpha)\cos\alpha x\, d\alpha = 0;\ a \le x \le \infty$$

Where:

$$G_1(\alpha) = [\gamma_1 (b_2 f_3 - b_3 f_2) \times \{g_1 k_1(\alpha) - g_2 k_2(\alpha) - g_3 k_3(\alpha)\} + \gamma_2 (b_3 f_1 - b_1 f_3) \times \{-g_1 l_1(\alpha) + g_2 l_2(\alpha) - g_3 l_3(\alpha)\}]/\{d\, F(\alpha)\}. \qquad (27)$$

$$d = b_1 (f_2 - f_3) + b_2 (f_3 - f_1) + b_3 (f_3 - f_1) \qquad (28)$$

Solution of the integral equation is given by:

$$\mathrm{C}(\alpha) = \frac{\pi a^2}{2F_1}\int_0^1 \xi^{1/2}\, \Phi(\xi) J_0(a\alpha\xi)\, d\xi, \qquad (29)$$

Where:

$$F_1 = \frac{1}{d}[\gamma_1 g_1 (b_2 f_3 - b_3 f_2) + \gamma_2 g_2 (b_3 f_1 - b_1 f_3) + \gamma_3 g_3 (b_1 f_2 - b_2 f_1)] \qquad (30)$$

and $J_0(a\alpha\xi)$ is the zero order Bessel function of first kind.

And, $\Phi(\xi)$ satisfies the integral equation:

$$\Phi(\xi) + \int_0^1 K(\xi,\eta)\Phi(\eta)\, d\eta = \begin{cases} -\sigma_h \xi^{1/2}, & \xi < \dfrac{c}{a} \\ -\sigma_h \xi^{1/2} + \dfrac{1}{2}\xi^{5/2}\dfrac{a^2}{c^2}\sigma_{ye} \times \left[1 - \dfrac{2}{\pi}\sin^{-1}\left(\dfrac{c/a}{\xi}\right) + \dfrac{1}{\pi}\sin\left\{2\sin^{-1}\left(\dfrac{c/a}{\xi}\right)\right\}\right], & \dfrac{c}{a} < \xi < 1 \end{cases} \qquad (31)$$

with kernel:

$$K(\xi,\eta) = \sqrt{\xi\eta}\int_0^\infty \left[\frac{G_1(\alpha/a)}{F_1} - 1\right]\alpha J_0(\alpha\xi) J_0(\alpha\eta)\, d\alpha, \qquad (32)$$

The constants a_h and b_h are obtained using the boundary conditions on the edges of the strip which for this case may be written as $\sigma_{zz}(x,h) = \sigma_h = \dfrac{\overline{c_{33}}}{c_{33}}\sigma_0 - \dfrac{e_{33}}{\varepsilon_{33}} D_0$ and $D_z(x,h) = D_0$ for $0 \le x \le \infty$.

Combining these with appropriate equation of equation (14), one obtains:

$$a_h = (\varepsilon_{33}\sigma_h + e_{33} D_0)/(c_{33}\varepsilon_{33} + e^2_{33}), \qquad (33)$$

$$b_h = -(e_{33}\sigma_h - c_{33} D_0)/(c_{33}\varepsilon_{33} + e^2_{33}). \qquad (34)$$

Thus displacement components and electric potential for this case are completely determined.

5.2. Case -II

Barring the edge conditions on the strip all other conditions remain the same as in the Case-I. Consequently solution obtained in 5.1. Case-I is valid from equations (18-32) for this case also.

The constants a_h and b_h are determined using the boundary condition on the edges of the strip which for the following case are $\sigma_{zz}(x,h) = \sigma_h = \sigma_0 - e_{33} E_0$ and $E_z(x,h) = E_0$ for $0 \le x < \infty$

Substituting these in equation (14) one obtains $a_h = (\sigma_h + e_{33} E_0)$ and $b_h = E_0$.

Hence the Case-II is solved too. Applications of the above analysis to calculate the quantities of interest is shown in section

6. Stress intensity factor, yield zone, crack opening displacement and crack growth rate

Opening mode stress intensity factor at the tip x=a is calculated using equation(31, 32, 14 and 15) and one finally obtains

$$K_I(a) = -\sqrt{\pi a}\, \Phi(1), \qquad (35)$$

Yield zone length is determined from the conditions that the stresses remain finite at every point of the strip, which gives the equation

$$\frac{c}{a} = \cos\left[\pi\left(\frac{c}{a}\right)^2 \frac{\sigma_h + U(h/a)}{\sigma_{ye}} - \frac{c}{a}\sqrt{1 - \left(\frac{c}{a}\right)^2}\right], \qquad (36)$$

with $\mathrm{U}(h/a) = \int_0^1 \mathrm{K}(1,\eta)\Phi(\eta)\, d\eta$. ()

Crack opening displacement at any point x on the rim of the crack is obtained by the superposition of the displacement due to the edge loading condition and displacement due to stress acting at the yield zones. These calculations finally lead to

$$COD(x)=$$
$$\frac{8\sigma_{ye}}{\pi\ F_1}\left[\frac{\sqrt{a^2-x^2}}{2}\left\{\left(\frac{a}{c}\right)^2\cos^{-1}\left(\frac{c}{a}\right)+\sqrt{\left(\frac{a}{c}\right)^2-1}\right\}\right.$$
$$-\frac{\pi}{4c^2}\int_x^a\frac{\alpha^3}{\sqrt{\alpha^2-x^2}}\left\{1-\frac{2}{\pi}\sin^{-1}\left(\frac{c}{\alpha}\right)\right.$$
$$\left.\left.+\ \frac{1}{\pi}\sin\left(2\sin^{-1}\left(\frac{c}{\alpha}\right)\right)\right\}d\alpha\right]. \tag{38}$$

Crack growth rate per cycle, dc/dN , is calculated from the fact that due to the process of loading, reloading and subsequent load cycles, the stresses and displacements under cyclic loading differs from monotonic loading but the solution found can be used as cyclic loading by making use of the following

$$\sigma_h \to \frac{\Delta\sigma}{2}, \sigma_0 \to \frac{\Delta\sigma_0}{2}, \sigma_{ye} \to \sigma_{yc}, \Delta\sigma=\sigma_2-\sigma_1$$

is applied tensile range, Δ denotes undergoing cyclic loading, σ_{yc} is the cyclic yield strength.

Crack growth rate is determined from

$$\frac{dc}{dN}=\frac{\pi}{192\gamma F_1\sigma^2_{yc}}(\Delta K_I)^4, \tag{39}$$

where:

$$\Delta K_I=\{\Delta\sigma+2U(h/a)\}\sqrt{\pi c}\ ,$$

$$\Delta\sigma=\begin{cases}\dfrac{\overline{c_{33}}}{c_{33}}\Delta\sigma_0-\dfrac{2e_{33}}{\varepsilon_{33}}D_0, & (Case-I)\\ \Delta\sigma_0-2e_{33}E_0, & (Case-II)\end{cases}. \tag{40}$$

7. Numerical results and discussion

Case study has been done for the ceramic PZT-4, PZT- 5H and $BaTiO_3$ studying variation of normalized stress intensity factor with respect to affecting parameter width of the strip, yield zone, crack length and material constants. Listing the material constants for piezoelectric ceramics is shown in Table 1.

Figure 2, shows the variation of normalized stress intensity factor's fourth power variation (which is directly proportional to crack growth rate) with respect to strip width to crack length ratio. For Case-I, it is observed that crack growth decreases in all ceramics when width of the strip is increased. It is also observed for negative to zero values to material constant the variation of stress intensity factor remains sort of parabolic but for higher positive value of the material constant the variation explicitly shows on exponentially decreasing behavior. Same is true for the Case-II. For Case-II the same variation is plotted in Figure 3 for the different values of the constant $e_{33}E_0/\sigma_0$. It may be noted that for this case the behavior of PZT-5H lying above PZT- 4 and below $BaTiO_3$ for all the values of $e_{33}E_0/\sigma_0$ varying from -0.25 to 0.5 while in Figure 2, shows that the curve for PZT-4 lying above $BaTiO_3$ and below PZT-5H for all the values of $c_{33}e_{33}D_0/\sigma_0\overline{c_{33}}\,\varepsilon_{33}$ varying from -0.25 to 0.5 .

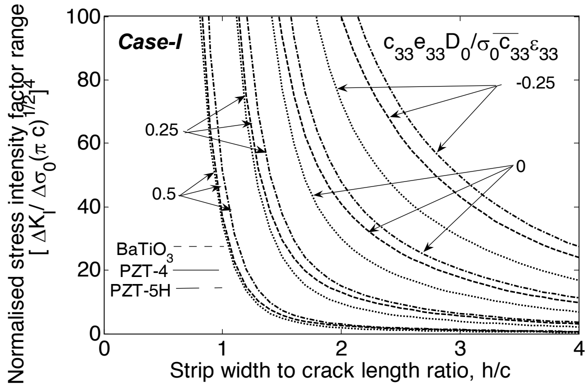

Fig. 2. Depicting a qualitative behavior of crack propagation rate at the tip of the crack versus h/c for in Case-I

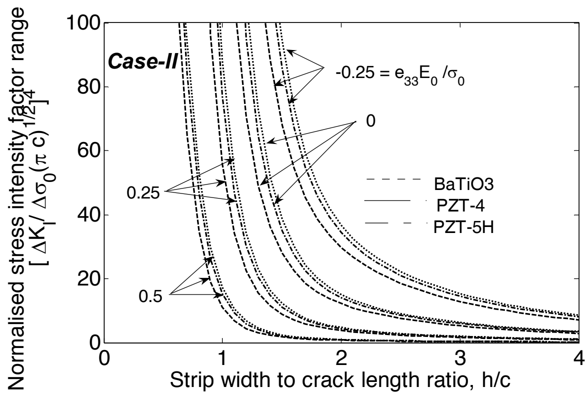

Fig. 3. Qualitative behavior of crack propagation rate at the tip of the crack versus h/c in Case-II

Figures 4 and 5 depict the variation of crack growth rate with respect to yield zone to crack length ratio for Case-I and II respectively. In both the cases as yield zone length increases the crack growth rate increases non-linearly. Ceramic~wise the same behavior is exhibited as in the case of Figures 2 and 3.

Table 1.
Listing the material constants for piezoelectric ceramics

Material constants	Ceramics		
	PZT-4	$BaTiO_3$	PZT-5H
c_{11} (10^{10}N/m^2)	13.9	15.0	12.6
c_{13} (10^{10}N/m^2)	7.43	6.60	5.30
c_{33} (10^{10}N/m^2)	11.5	14.6	11.7
c_{44} (10^{10}N/m^2)	2.56	4.4	3.53
e_{13} (C/m^2)	- 5.2	- 4.35	- 6.50
e_{33} (C/m^2)	15.1	17.5	23.3
e_{15} (C/m^2)	12.7	11.4	17.0
ε_{11} (10^{-10}C/Vm)	64.6	98.7	151
ε_{33} (10^{-10}C/Vm)	56.2	112	130

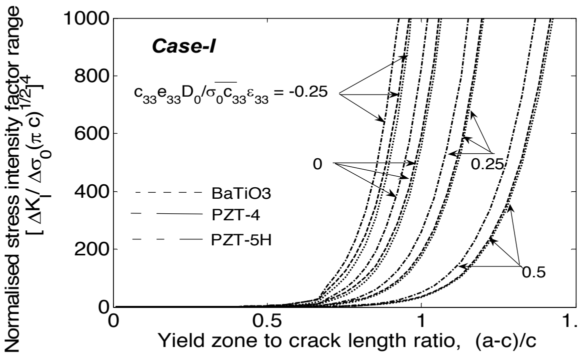

Fig. 4. Qualitative behavior of crack propagation rate at the tip of the crack versus (a-c)/c in Case-I

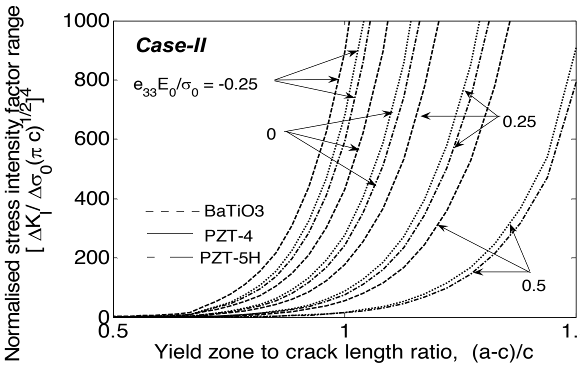

Fig. 5. Qualitative behavior of crack propagation rate at the tip of the crack versus (a-c)/c for Case-II

8. Conclusions

The present investigations porposes a crack opening arrest model for a poled narrow piezoelectric infinitely long strip. For the quantities of interest viz. yield zone length, crack face opening displacement and crack growth rate the analytic closed form expressions are derived and their qualitative behavior with respect to affecting parameters have been plotted and analyzed.

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References

[1] Y. Shindo, E. Ozawa, Singular stresses and electric fields of a cracked piezoelectric strip, International Journal of Applied Electromagnetic Materials 1 (1990) 77-87.
[2] Y. Shindo, E. Ozawa, Dynamic analysis of a cracked piezoelectric material, Mechanical model of new electromagnetic materials, Proceedings of the IUTAM Symposium, 1990, 297-304.
[3] V.V. Belyaev, Spall damage modeling and dynamic fracture specificities of ceramics, Journal of Materials Processing Technology 32 (1992) 135-144.
[4] Y. Shindo, K. Tanaka, F. Narita, Singular stress and electric fields of a piezoelectric ceramic strip with a finite crack under longitudinal shear, Acta Mechanica 120 (1997) 31-45.
[5] Y. Shindo, K. Tanaka, F. Narita, Electroelastic intensification near antiplane shear crack in orthotropic piezoelectric ceramic strip, Theoretical and Applied Fracture Mechanics 25 (1996) 65-71.
[6] Z.T. Chen, Crack tip field of an infinite piezoelectric strip under anti-plane impact, Mechanics Research Communications 25 (1998) 313-319.
[7] M.T. Hayajneh, V.P. Astakhov, M.O.M. Osman, An analytical evaluation of the cutting forces in orthogonal cutting using a dynamic model of shear zones with parallel boundaries, Journal of Materials Processing Technology 82 (1998) 61-67.
[8] F. Narita, Y. Shindo, Fatigue crack propagation in a piezoelectric ceramic strip subjected to mode III loading, Acta Mechanica 137 (1999) 55-63.
[9] Z.T. Chen, S.A. Meguid, The transient response of a piezoelectric strip with a vertical crack under electromechanical impact load, International Journal of Solids and Structures 37 (2000) 6051-6062.
[10] F. Narita, Y. Shindo, Mode I crack growth rate for yield strip model of a narrow piezoelectric ceramic body, Theoretical and Applied Fracture Mechanics 36 (2001) 73-85.
[11] X. Wang, S. Yu, Transient response of a crack in a piezoelectric strip subjected to the mechanical and electrical

impacts: mode-III problem, International Journal of Solids and Structures 37 (2000) 5795-5808.
[12] B.L. Wang, Y.W. Mai, A piezoelectric material strip with a crack perpendicular to its boundary surfaces, International Journal of Solids and Structures 39 (2002) 4501-4524.
[13] S. Li, On saturation-strip model of a permeable crack in a piezoelectric ceramic, Acta Mechanica 165 (2003) 47-71.
[14] J. Chen, Z. Liu, Z. Zou, The central crack problem for a functionally graded piezoelectric strip, International Journal of Fracture 121 (2003) 81-94.
[15] R.R. Bhargava, S. Hasan, Generalized Dugdale model solution for a piezoelectric plate weakened by two straight cracks, Proceedings of the International Conference "Advances in Materials and Processing Technologies", 2003, vol. 1, 647-650.
[16] M. Szutkowska, Fracture resistance behavior of Alumina-Zirconia composites, Proceedings of the International Conference "Advances in Materials and Processing Technologies", 2003, vol. 1, 1251-1254.
[17] V. Govorukha, M. Kamlah, D. Munz, The interface crack problem for a piezoelectric semi-infinite strip under concentrated electromechanical loading, Engineering Fracture Mechanics 71 (2004) 1853-1871.
[18] L. Ma, L.Z. Wu, Z.G. Zhou, L.C. Guo, Fracture analysis of a functionally graded piezoelectric strip, Composite structures 69 (2005) 294-300.
[19] R.R. Bhargava, N. Saxena, Solution for a cracked piezoelectric plate subjected to variable load on plastic zones under mode-I deformation, Journal of Materials Processing Technology 164-165 (2005) 1495-1499.
[20] S. Ueda, A finite crack in a semi-infinite strip of a graded piezoelectric material under electric loading, European Journal of Mechanics A25 (2006) 250-259.
[21] R.R. Bhargava, A. Setia, Crack arrest model for a piezoelectric strip subjected to Mode-I loadings, Journal of Achievements in Materials and Manufacturing Engineering 22/2 (2007) 9-22.